

Theory of Programming Languages

CMPT 331 • Fall 2018

—Fun with Mbnecb Dbmdvmvt - 100 points

Goal To appreciate the joy of the Lambda Calculus as a fundamental model of computation.

Problems *Each part of each problem is worth 10 points*¹.

1. Beta-reduce the following expressions to their normal form:

a. $(\lambda a \lambda y . y a) (z z)$

b. $(\lambda x \lambda y . (x y)) (\lambda z . y)$

c. $(\lambda x . (x x)) (\lambda y . (y y))$

d. $K x y$

e. $S K$

f. $(S K) y y z$

g. $K' y y z$

2. What is the normal form of $S (K S) (K I)$?

3. Prove the following equivalencies by reducing each side to its normal form.

a. $I = S K K$

b. $S K K = K I I$

4. Given the definition of Church numerals below, what does $(m n)$ do when m and n are Church numerals? For example $(\bar{2} \bar{3})$. It may be easier to work out as $\lambda m \lambda n . (m n)$. Show your work (or at least an example).

Church Numerals

Let $\bar{0} = \lambda f x . x$

Let $\bar{1} = \lambda f x . (f x)$

Let $\bar{2} = \lambda f x . (f (f x))$

Let $\bar{3} = \lambda f x . (f (f (f x)))$

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Let $\bar{n} = \lambda f x . (f^n . x)$

Let successor = $\overline{\text{succ}} = \lambda n f x . n f (f x)$

$(\overline{\text{succ}} \bar{0}) \equiv (\lambda n f x . n f (f x)) (\bar{0})$

$[\bar{0}/n] \text{ in } \lambda f x . n f (f x)$

$\rightarrow_{\beta} \lambda f x . \bar{0} f (f x)$

$[(f x)/f] \text{ in } \lambda x . \bar{0} f$

$\rightarrow_{\beta} \lambda x . \bar{0} (f x)$

$\equiv \lambda x . (\lambda f x . x) (f x)$

$[(f x)/x] \text{ in } \lambda f x . x$

$\rightarrow_{\beta} \lambda f x . (f x)$

$\equiv \bar{1}$

Submitting Write your summary using LaTeX. (See template and resources on our web site.) **Print** your document and hand it to me any time **before** the class in which it is due. Remember to include your name.

¹ Yes, I know it adds up to 110 points. You can make a mistake or two and still get a good score. Feel the love.