

The Bellman-Ford Single-Source Shortest Path Algorithm

CLRS chapter 24.1

Graph G

a weighted, directed graph with negative edge weights

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$G.E = (t, x : w = 5)$

$(t, y : w = 8)$

$(t, z : w = -4)$

$(x, t : w = -2)$

$(y, x : w = -3)$

$(y, z : w = 9)$

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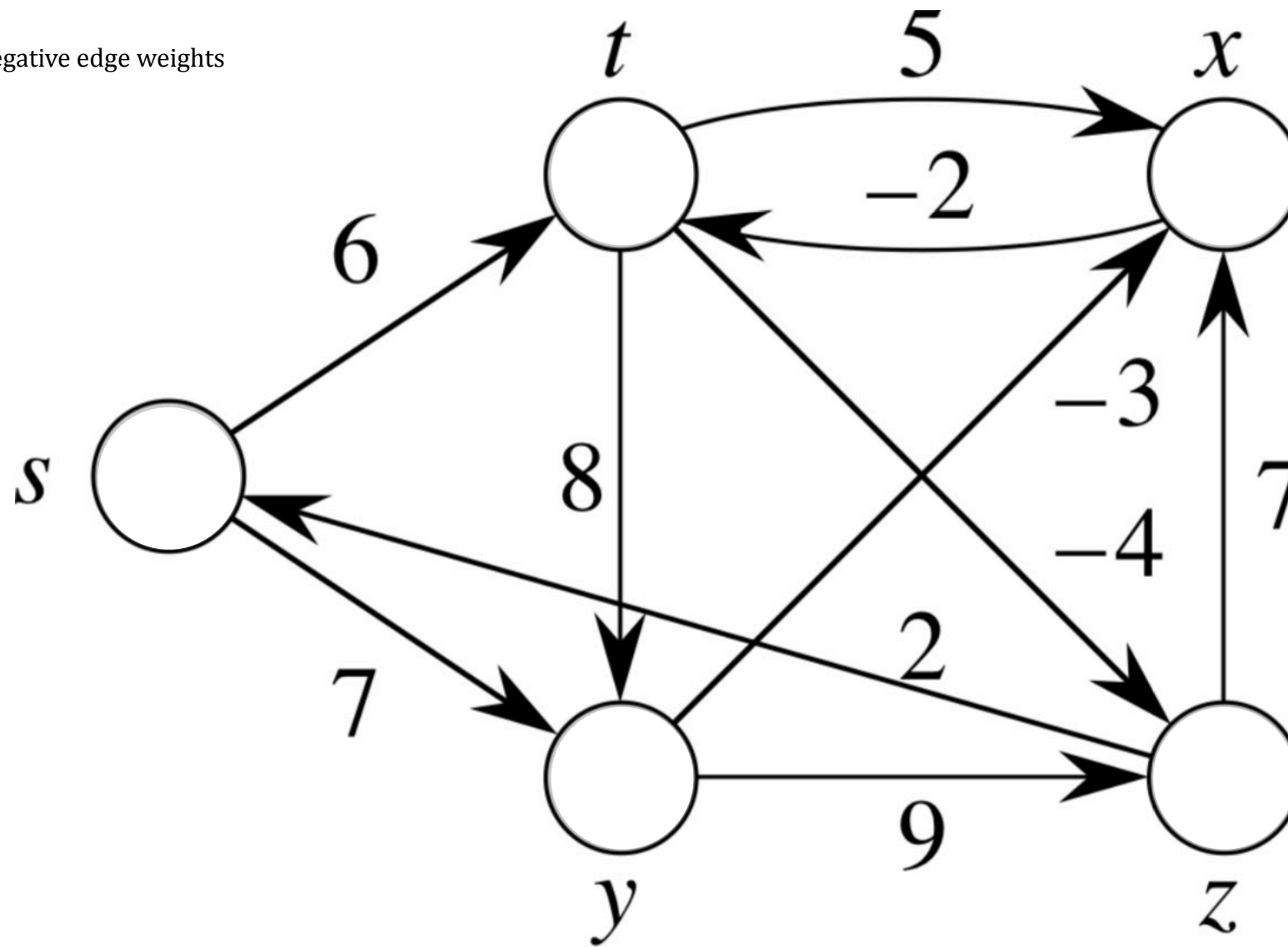
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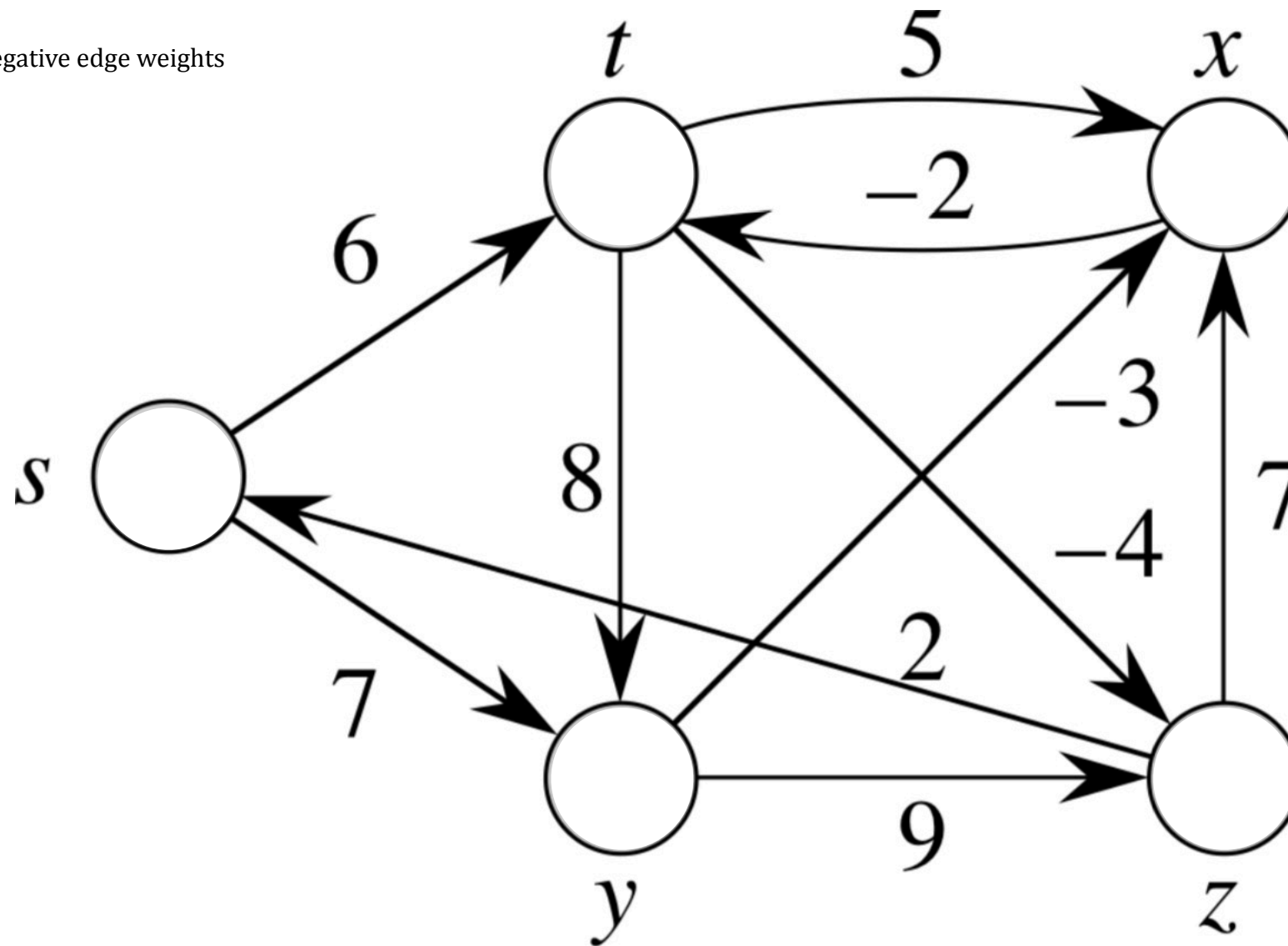
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BELLMAN-FORD(G, w, s)

 INIT-SINGLE-SOURCE(G, s)

for $i = 1$ to $|G.V| - 1$

for each edge $(u, v) \in G.E$

 RELAX(u, v, w)

for each edge $(u, v) \in G.E$

if $v.d > u.d + w(u, v)$

return FALSE

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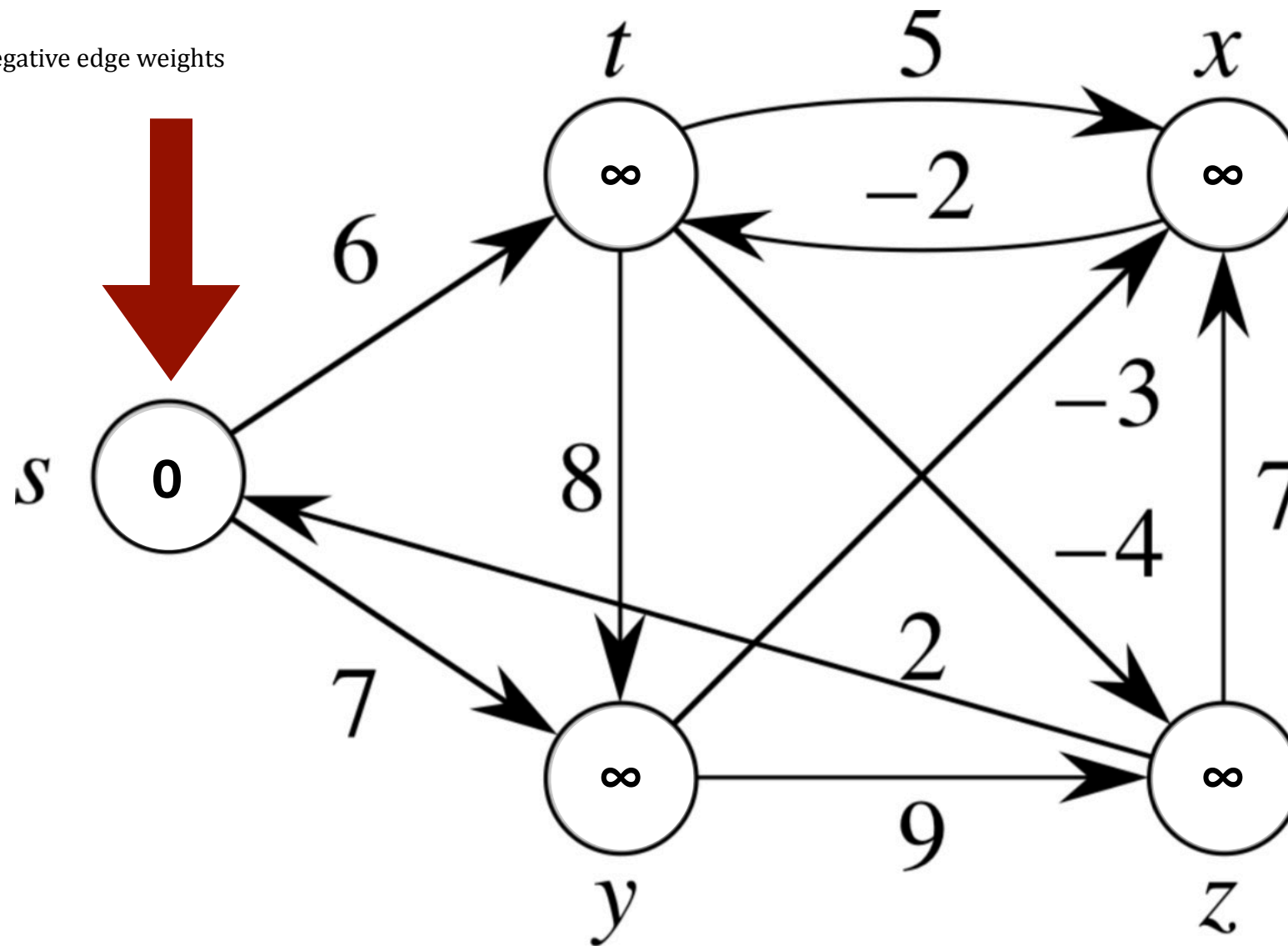
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INIT-SINGLE-SOURCE(G, s)

for each $v \in G.V$

$v.d = \infty$ // estimate of shortest path distance

$v.\pi = \text{NIL}$ // predecessor vertex

$s.d = 0$

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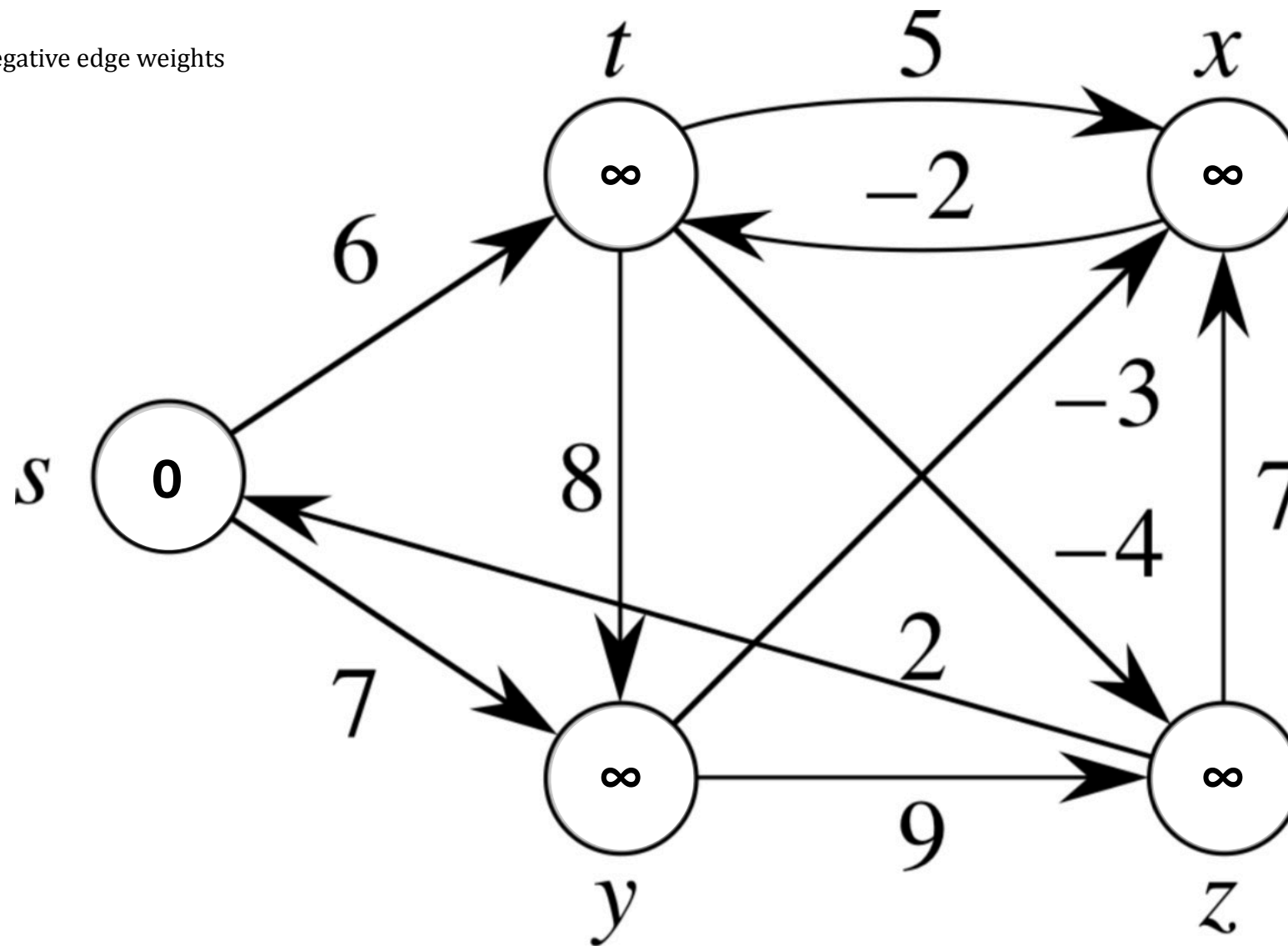
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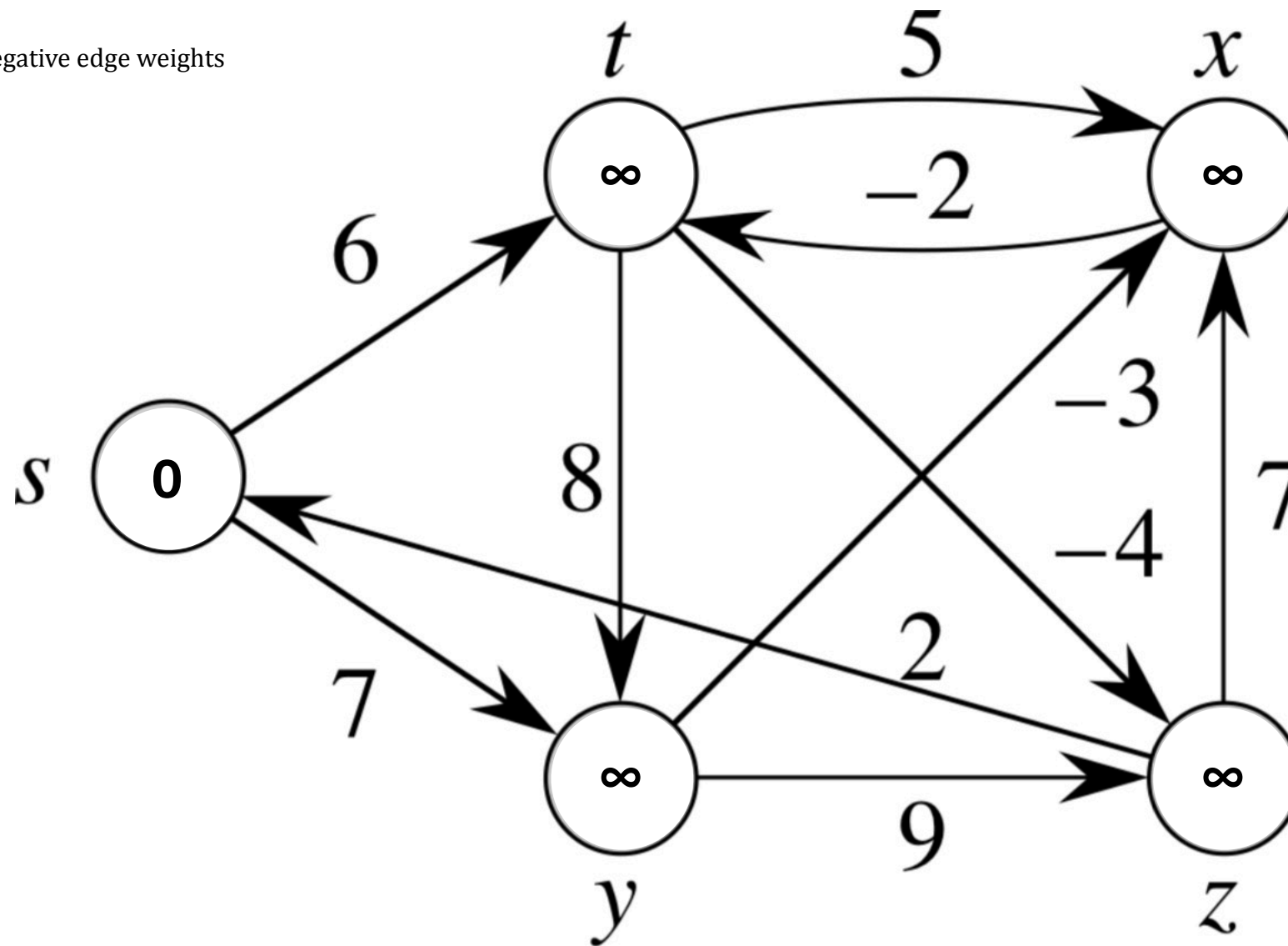
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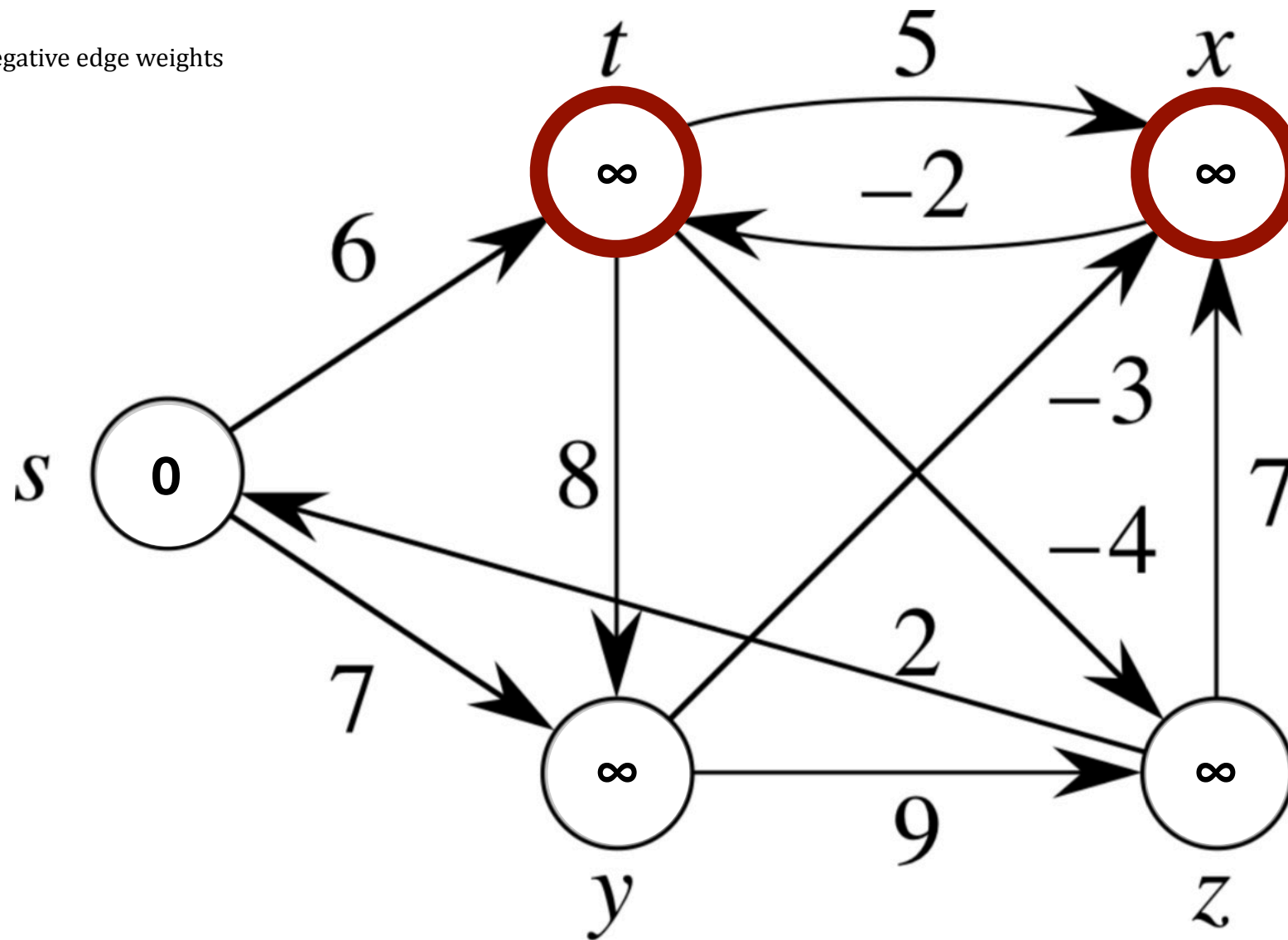
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if $v.d > u.d + w(u, v)$ // if $x.d(\infty) > t.d(\infty) + w(t,x)(5)$ then

$v.d = u.d + w(u, v)$ // set $x.d = t.d + w(t,x)$

$v.\pi = u$ // set predecessor vertex to t

Assume for any real number a : $a + \infty = \infty + a = \infty$

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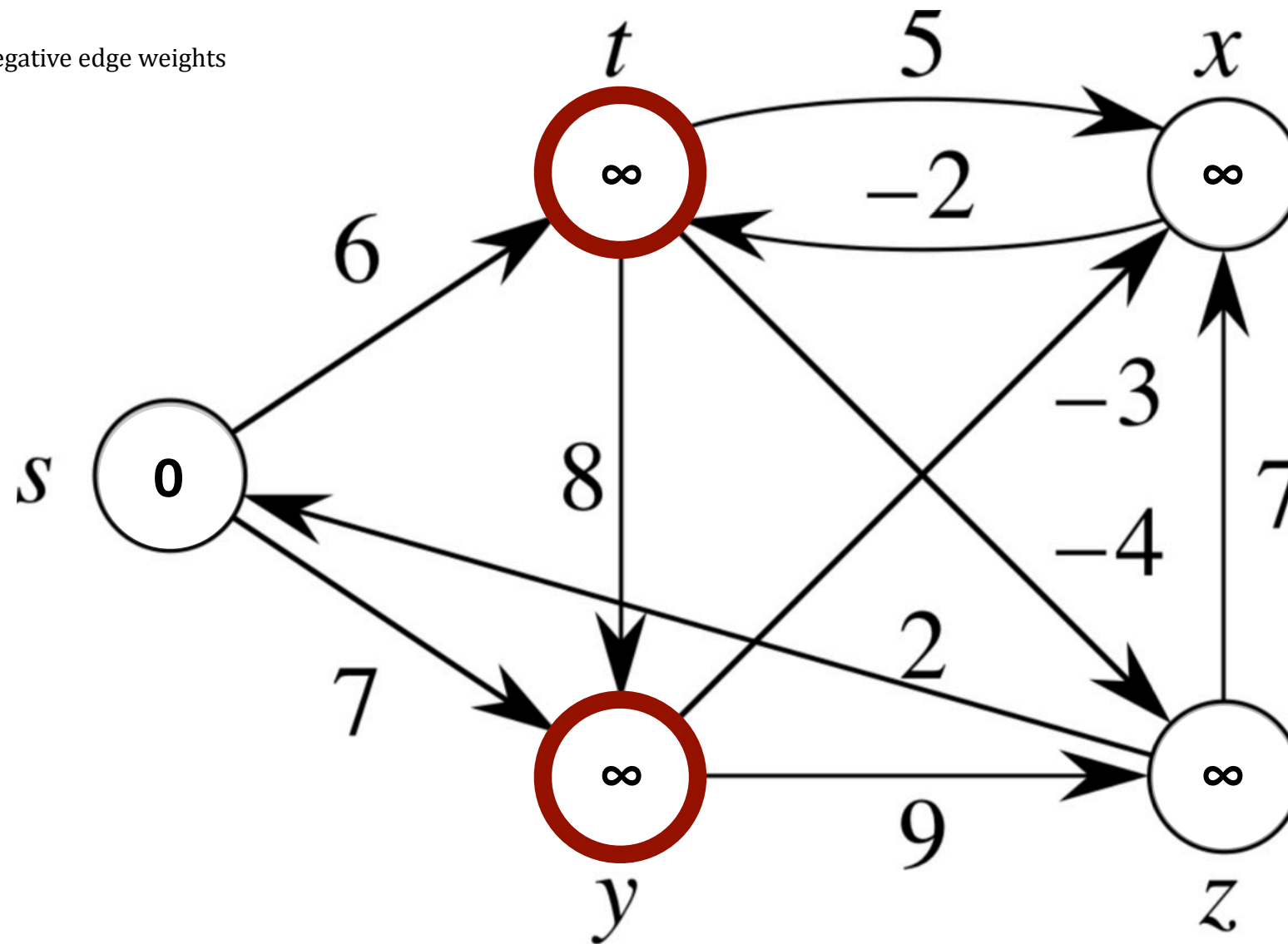
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 $v.d = u.d + w(u, v)$ // set $y.d = t.d + w(t,y)$
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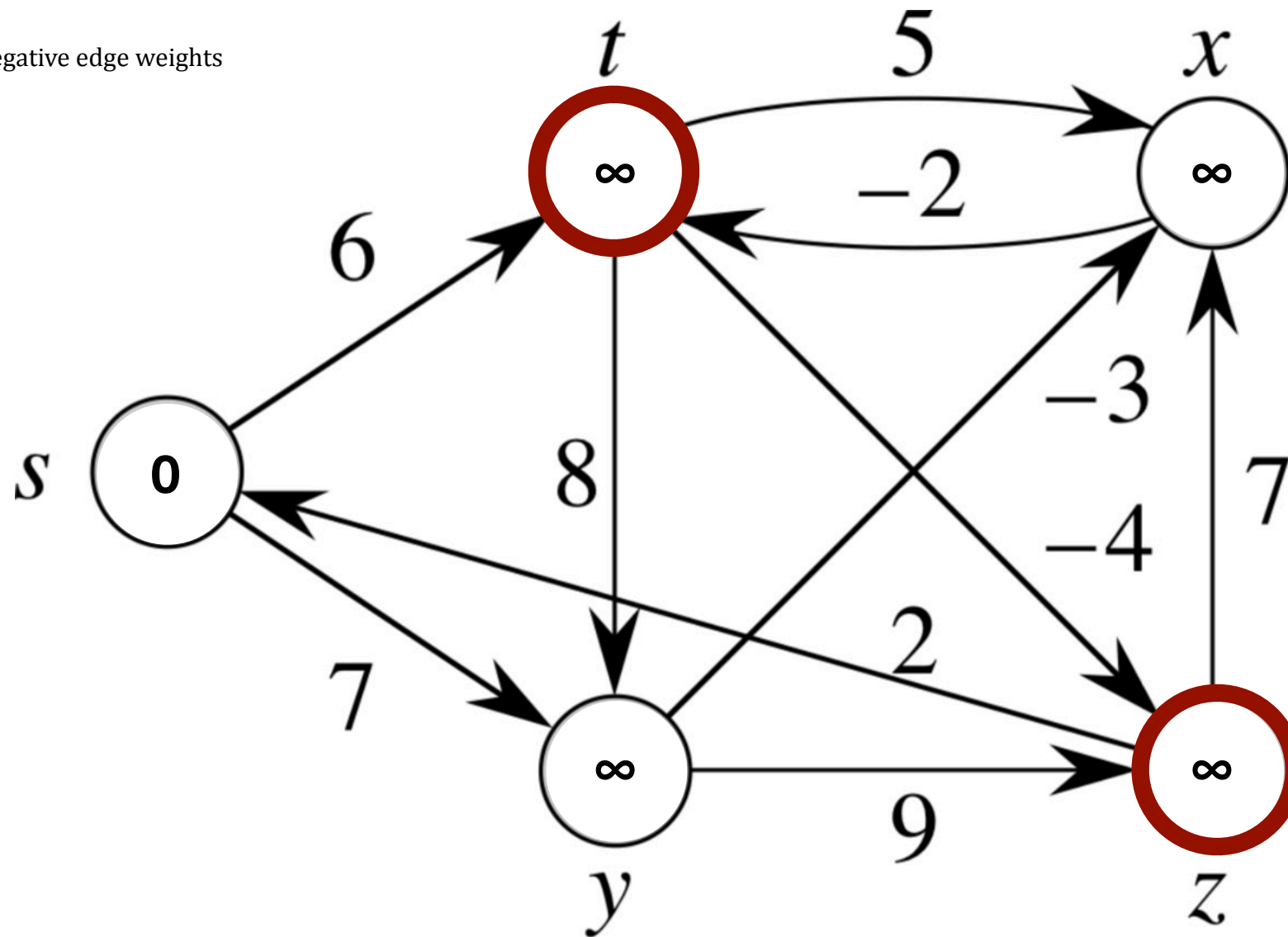
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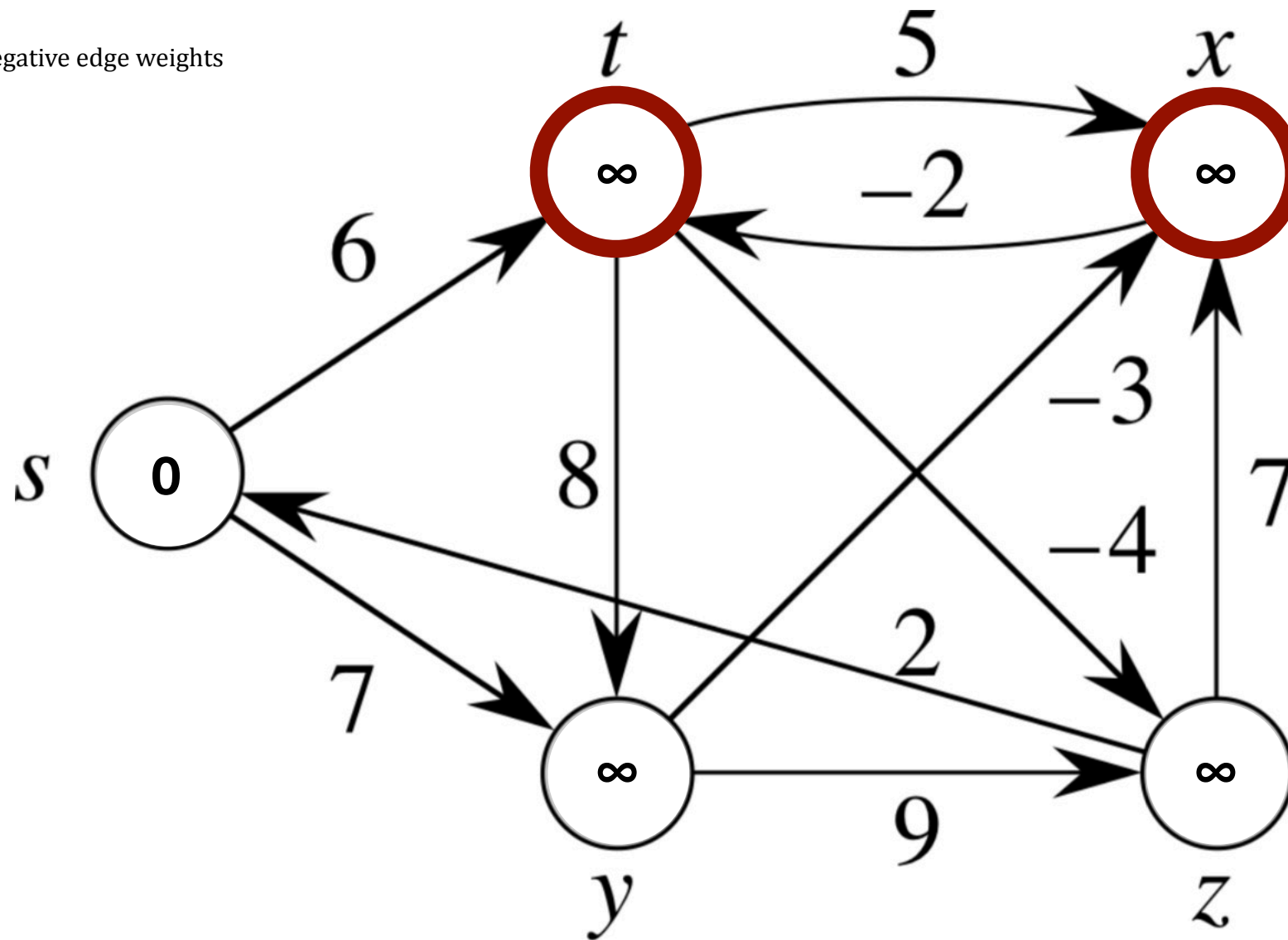
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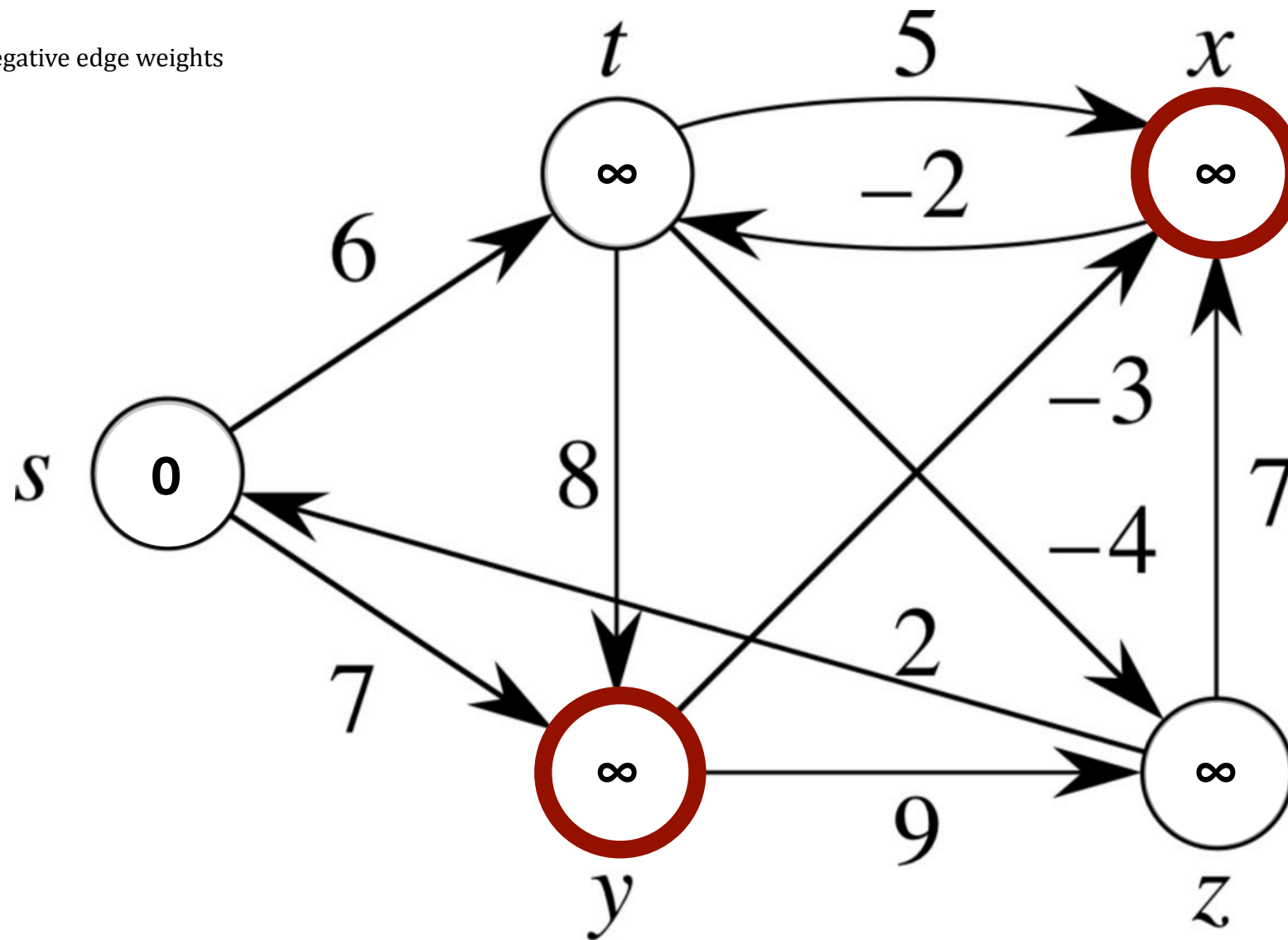
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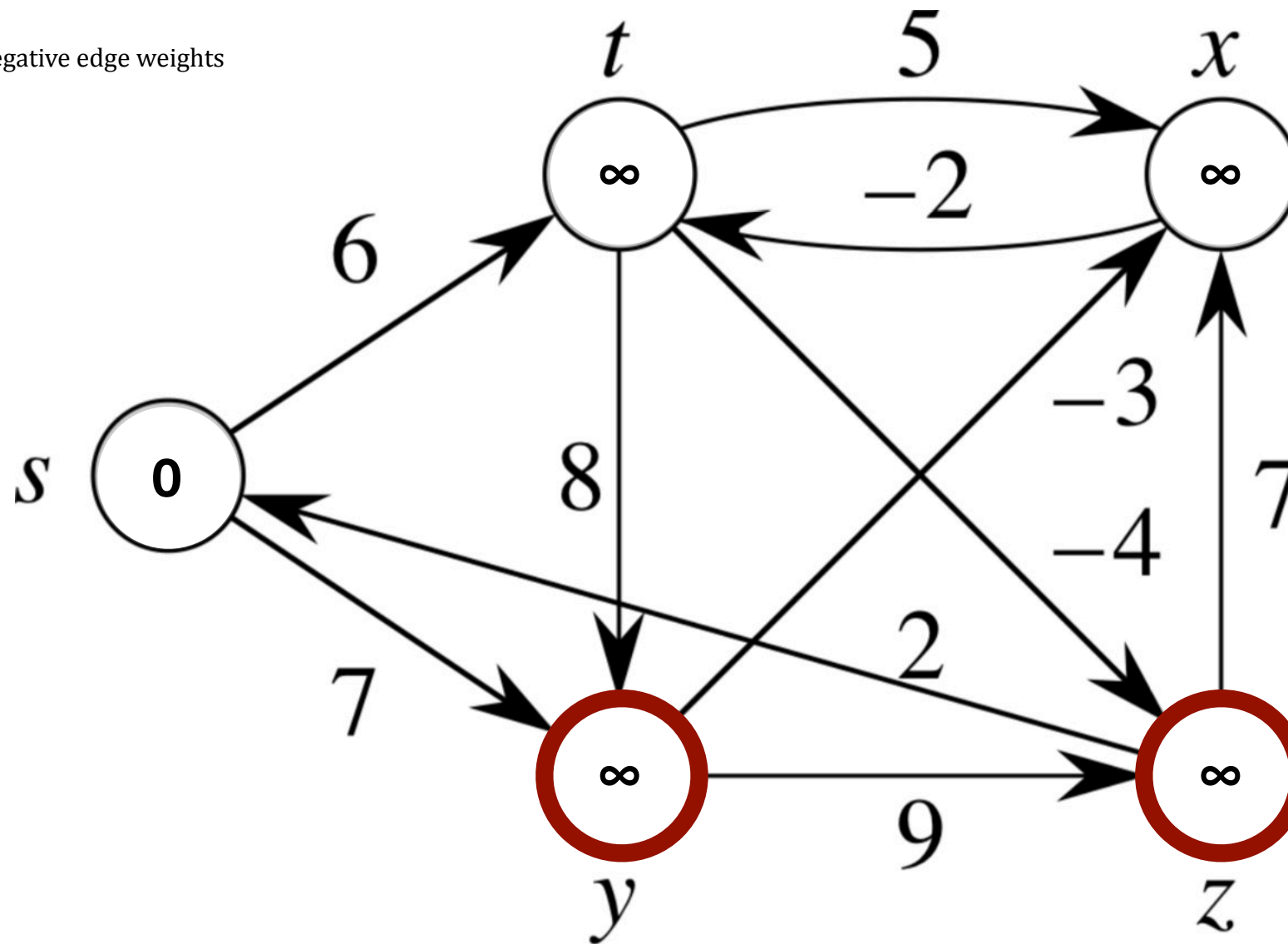
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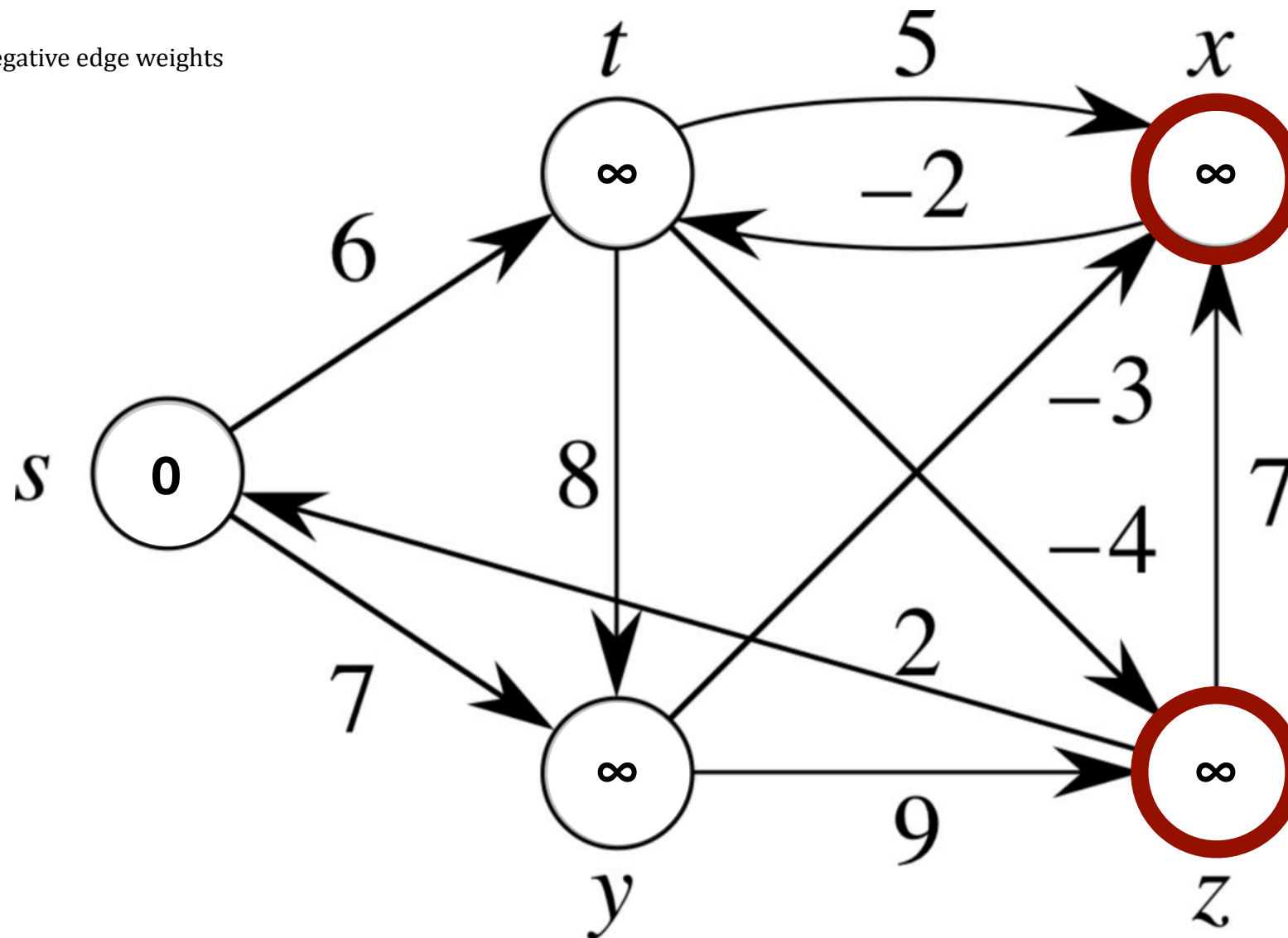
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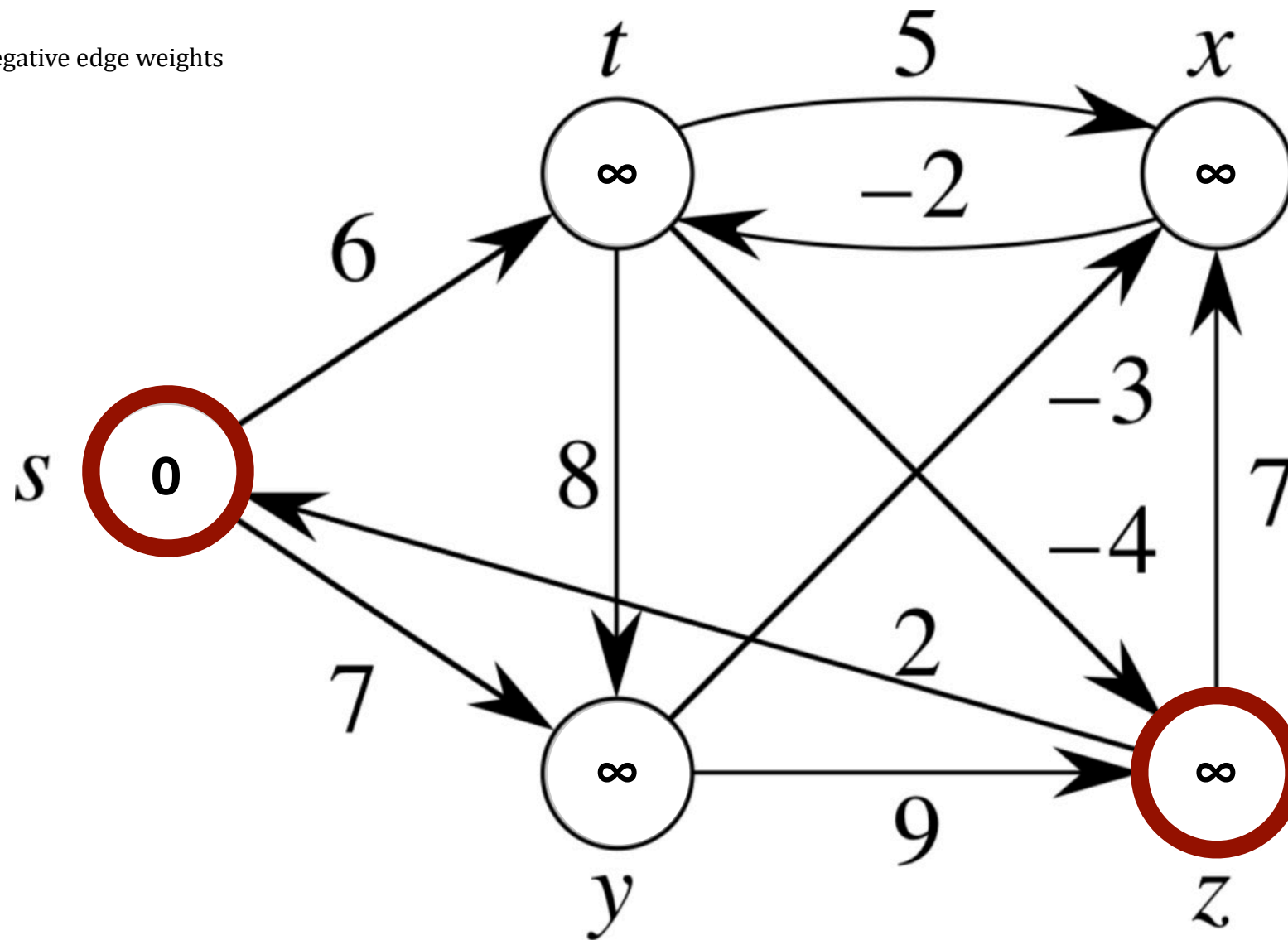
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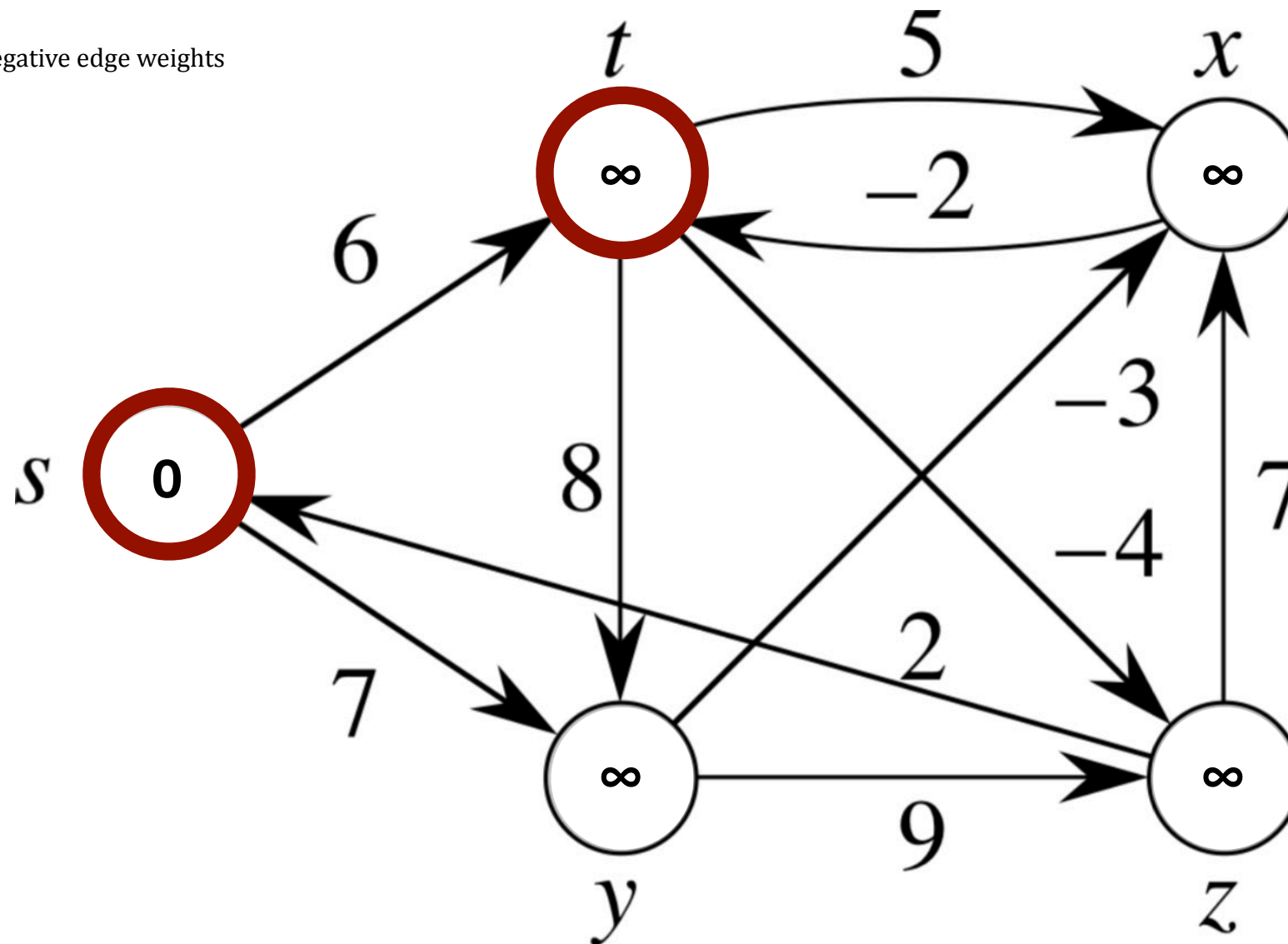
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$v.d = u.d + w(u, v)$ // set $t.d = s.d + w(s,t) = 6$

$v.\pi = u$ // set predecessor vertex to s

Finally!

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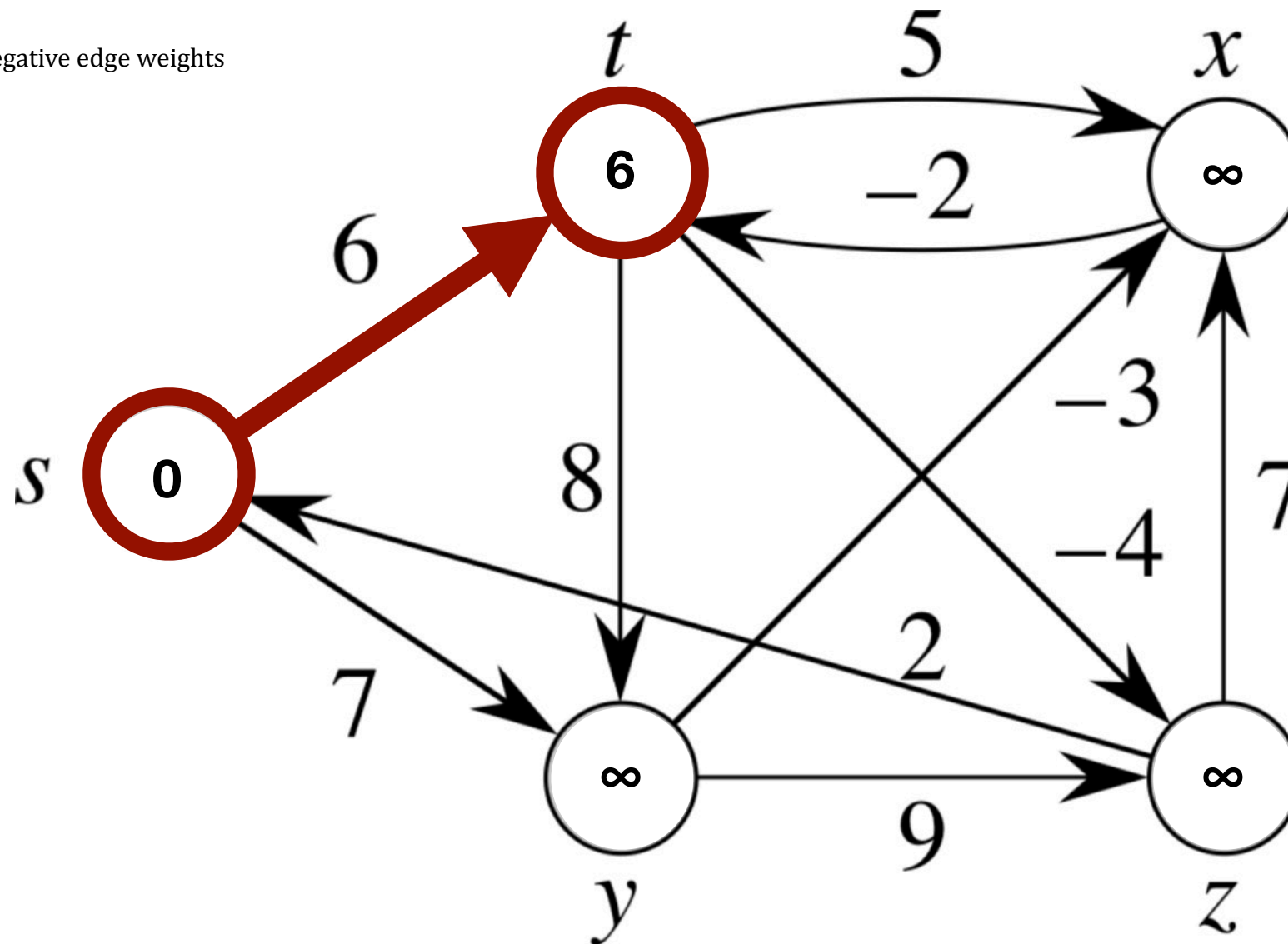
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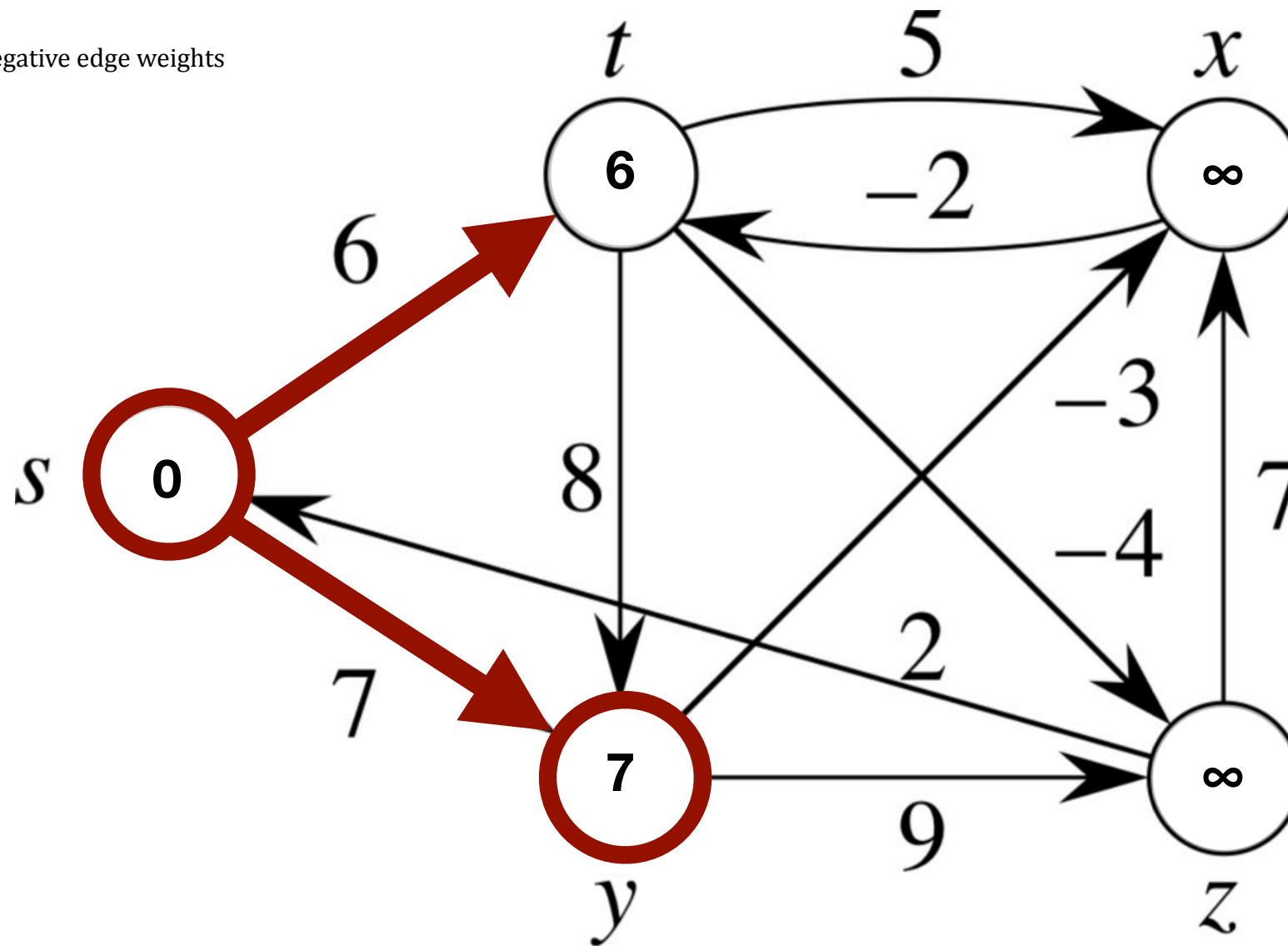
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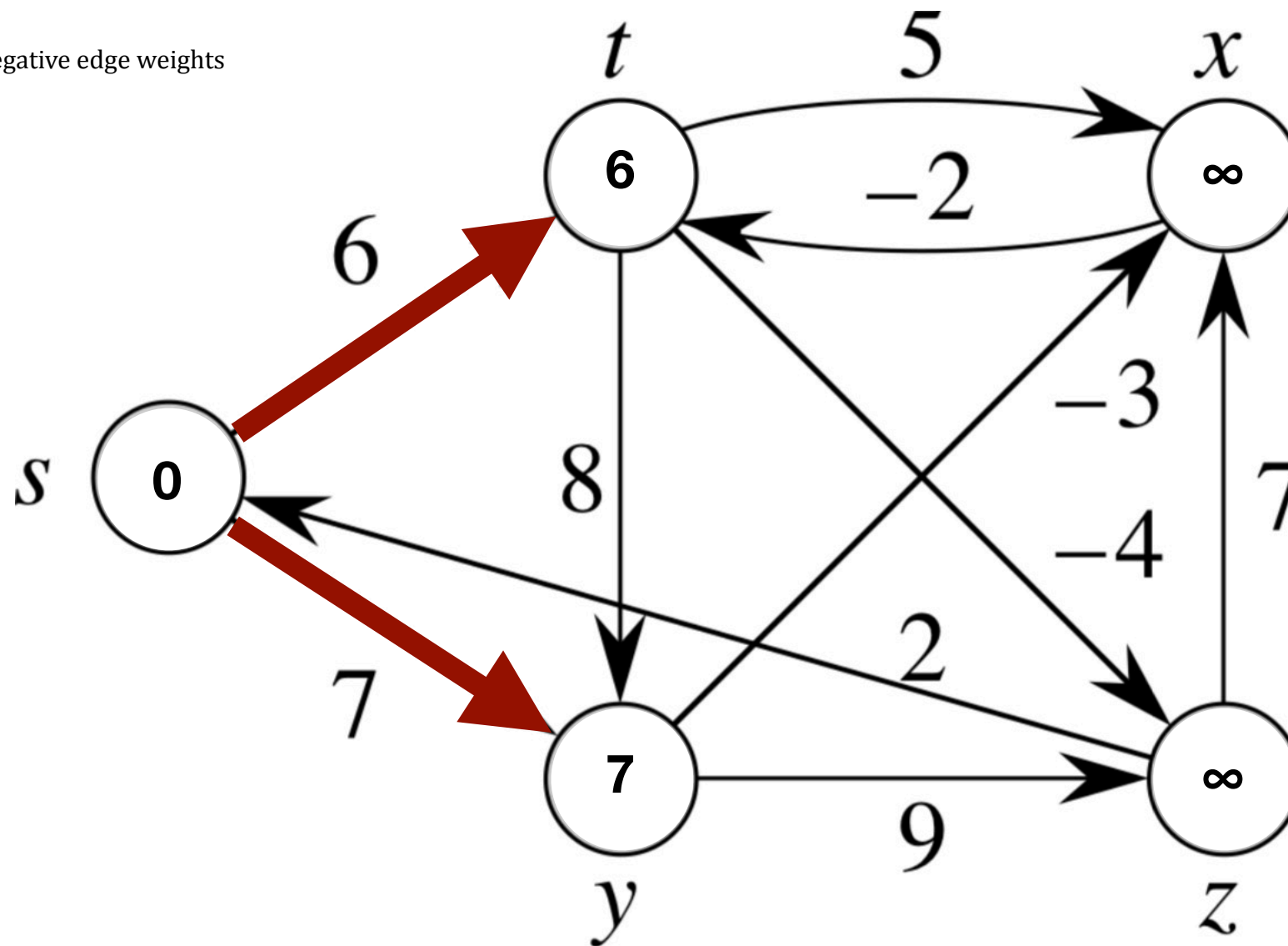
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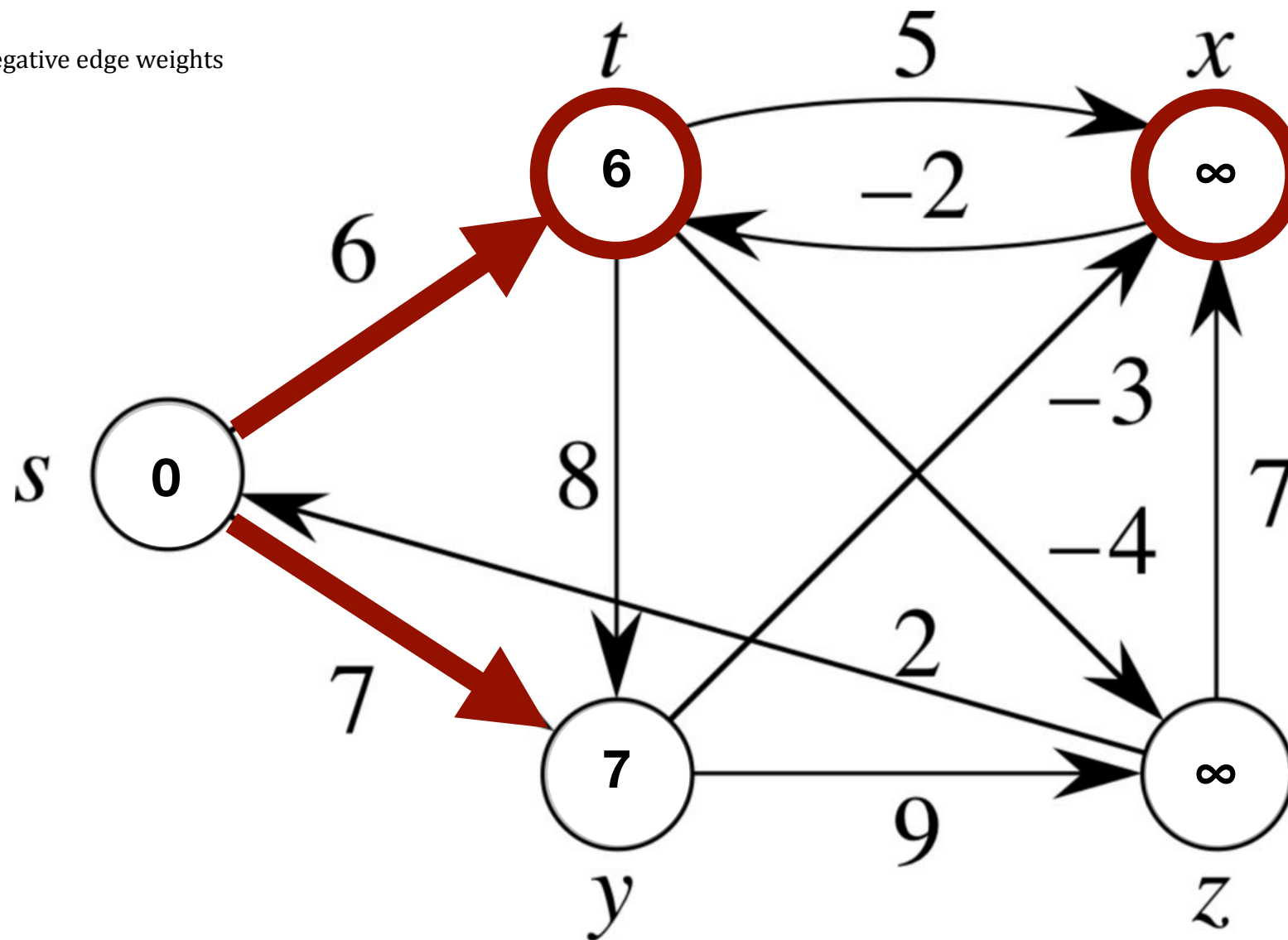
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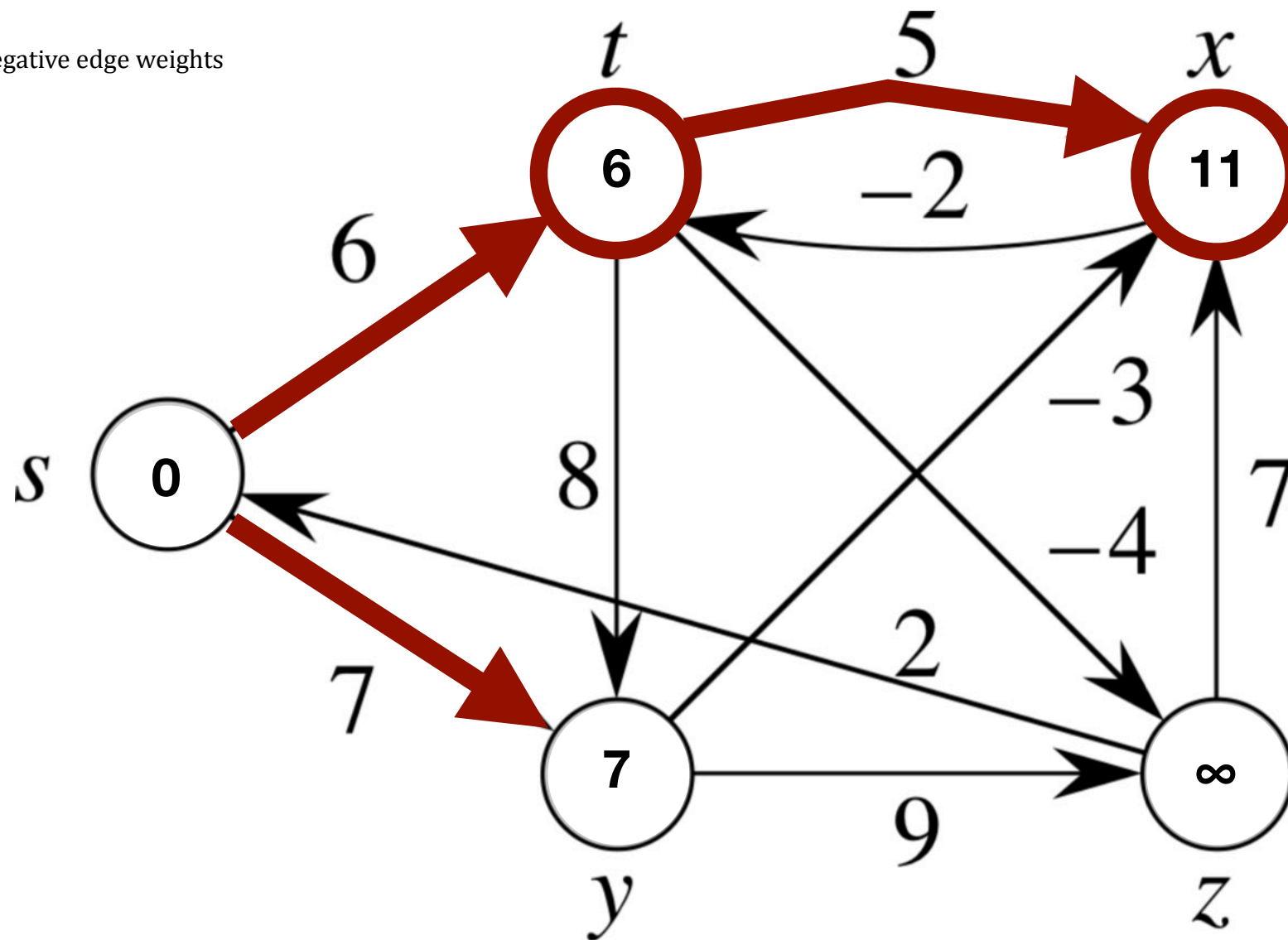
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Remember,
these are estimates
until we finish.
Can we do better?

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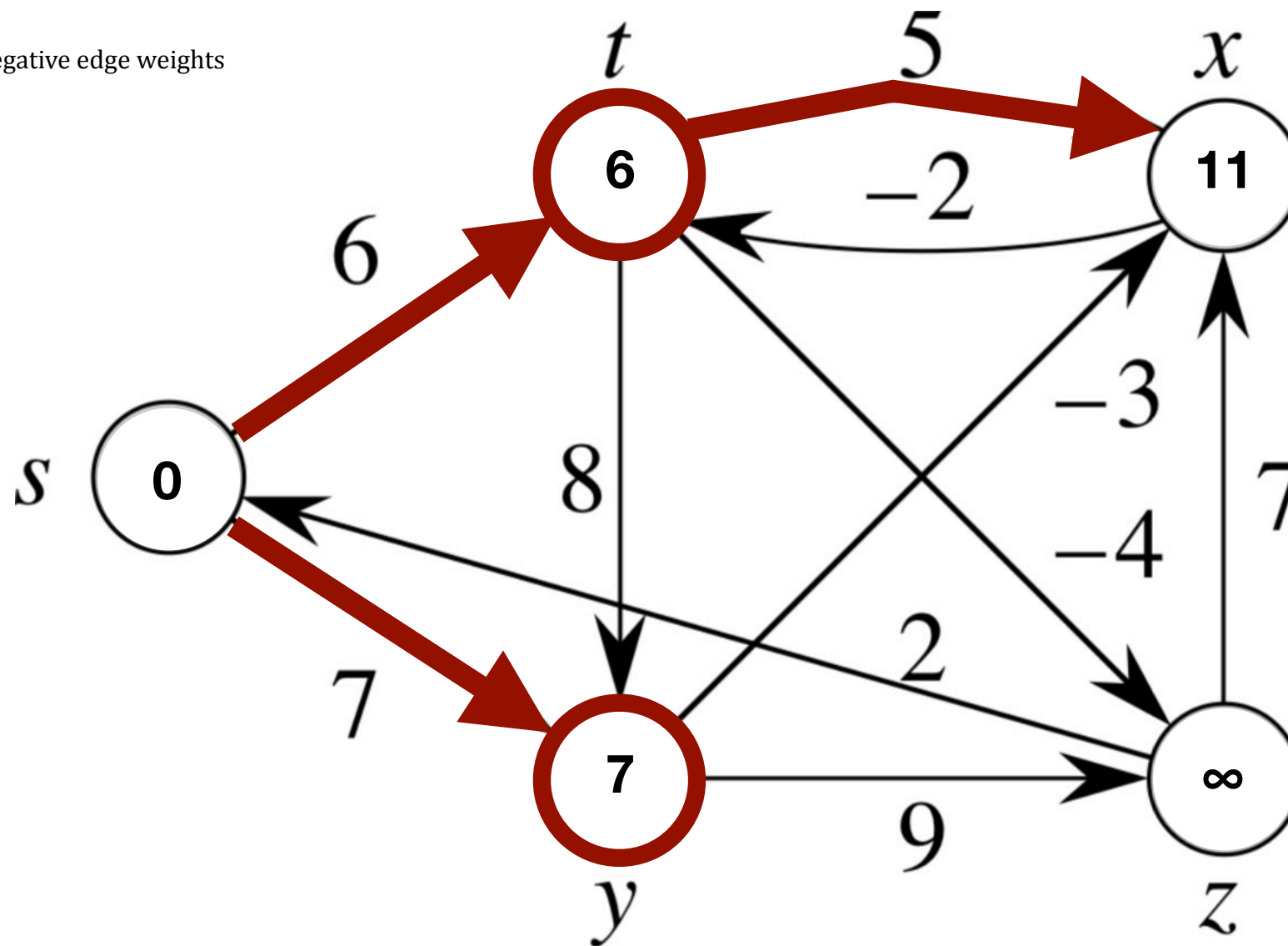
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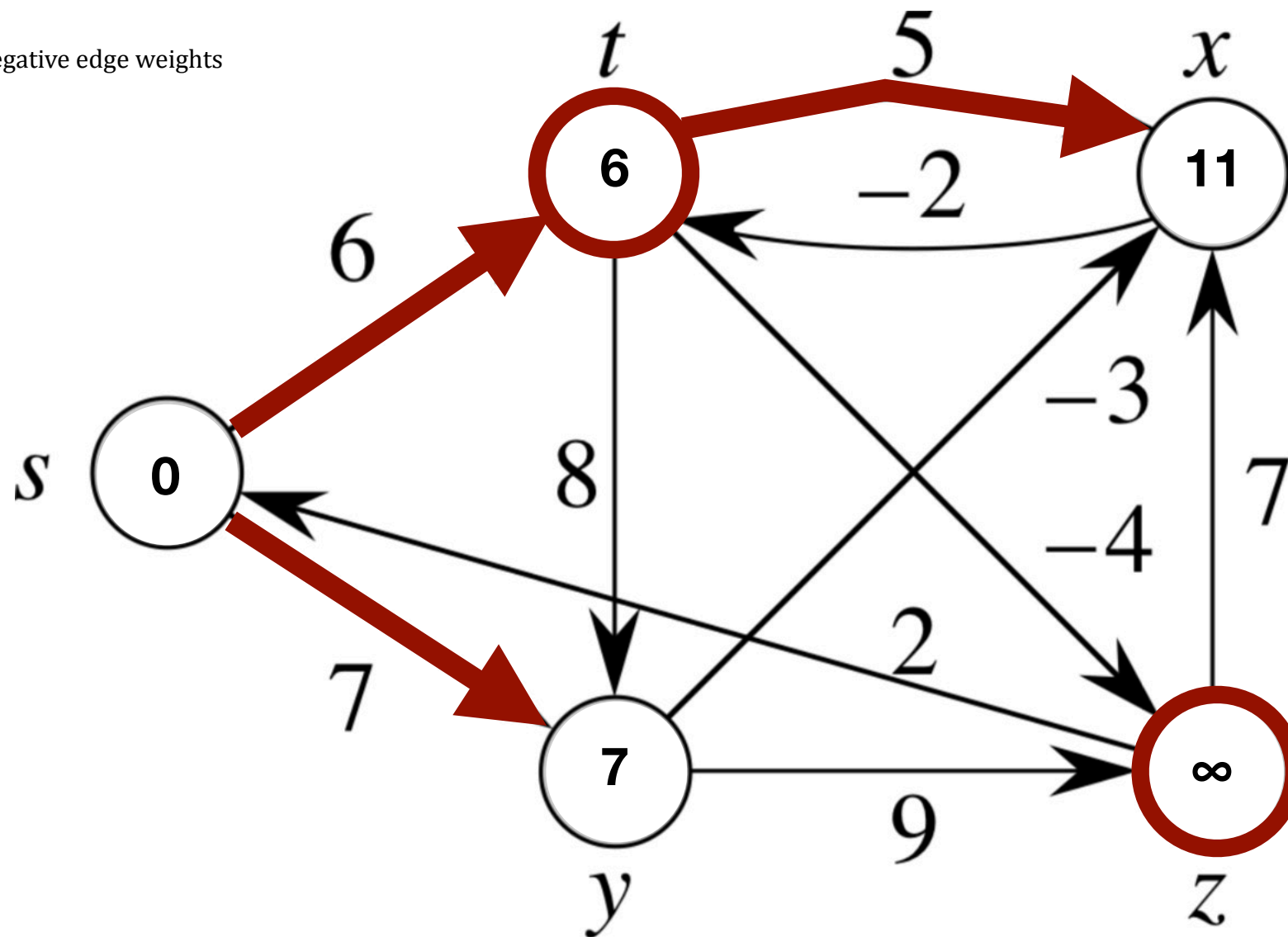
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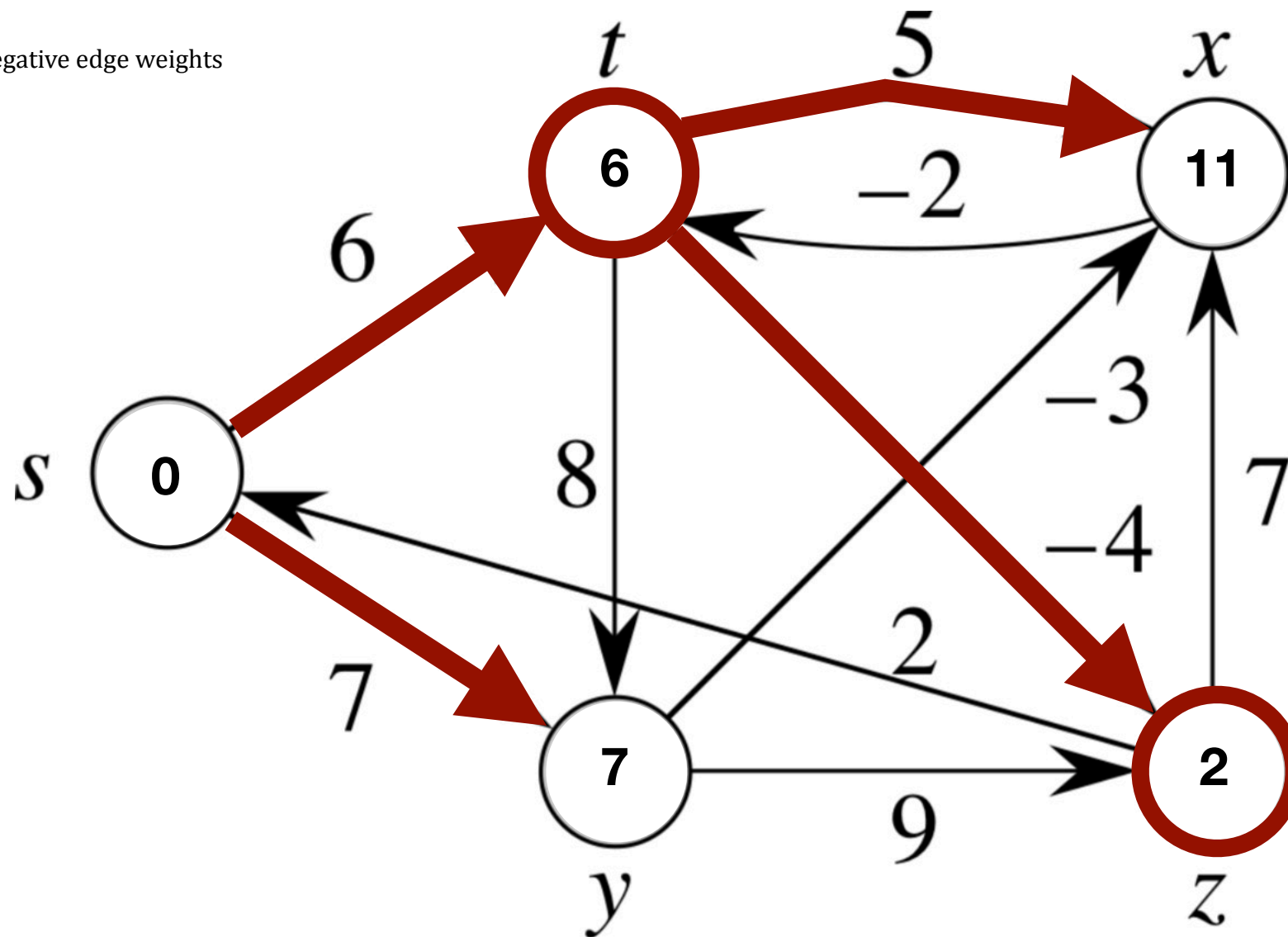
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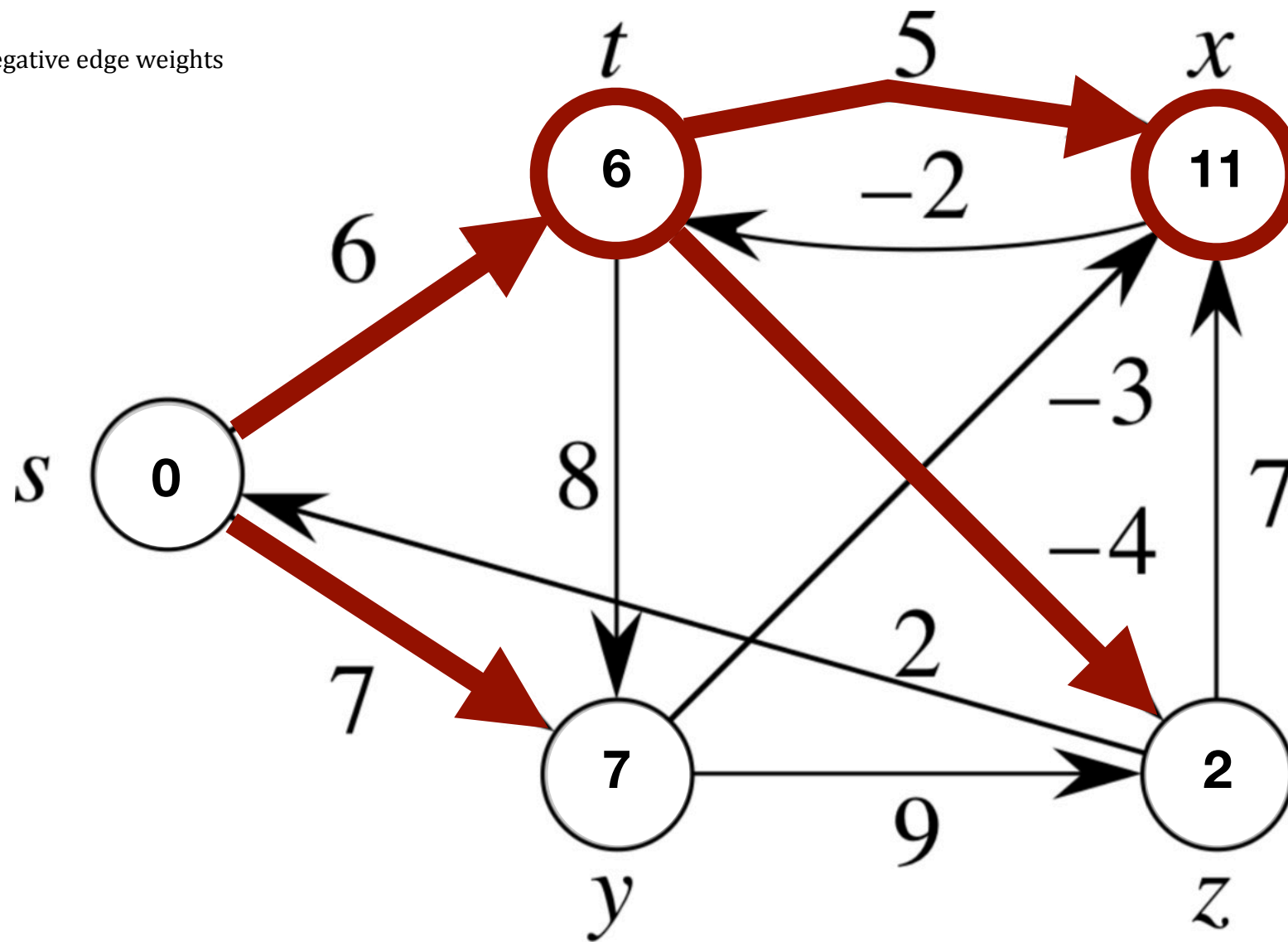
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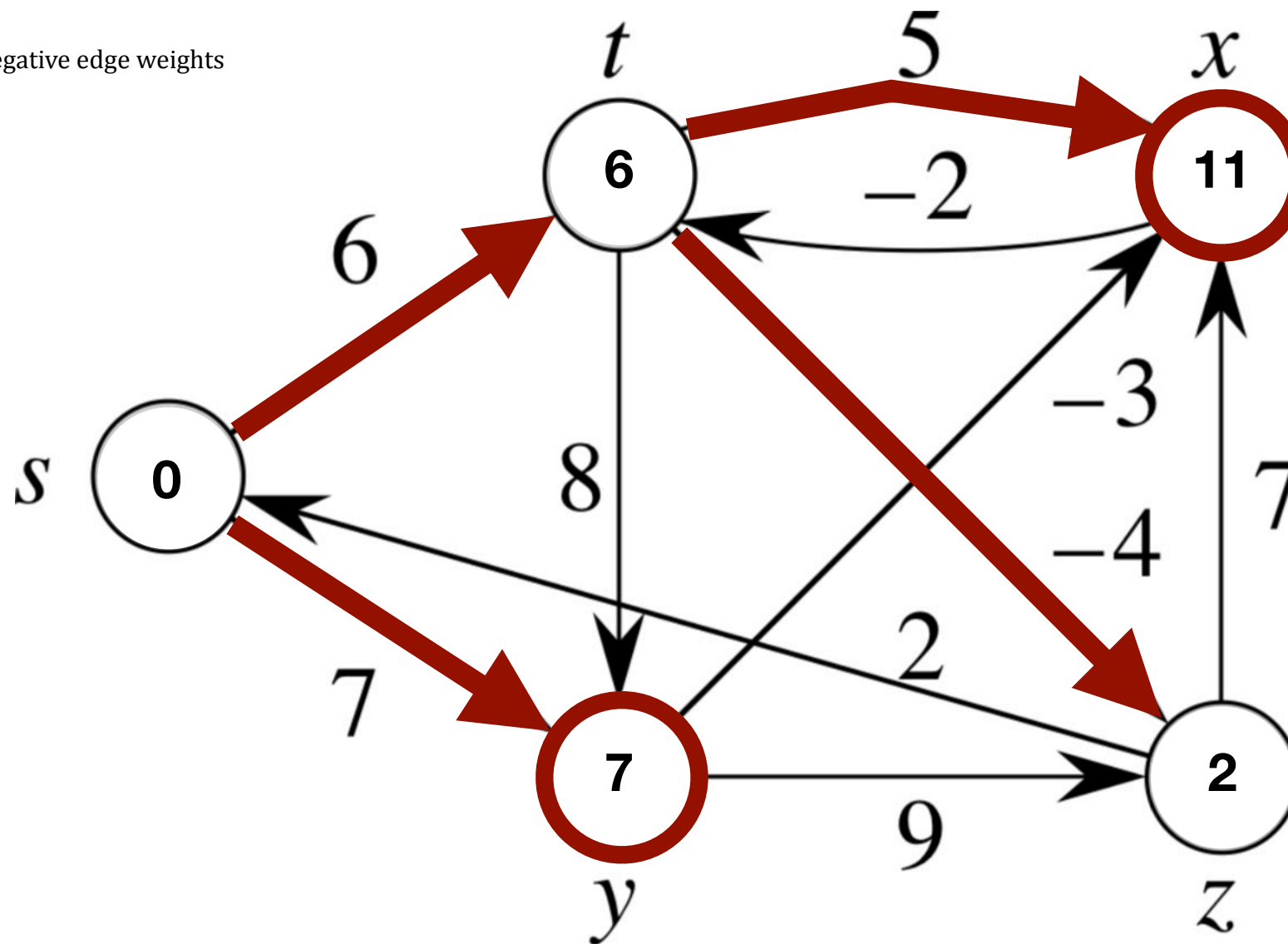
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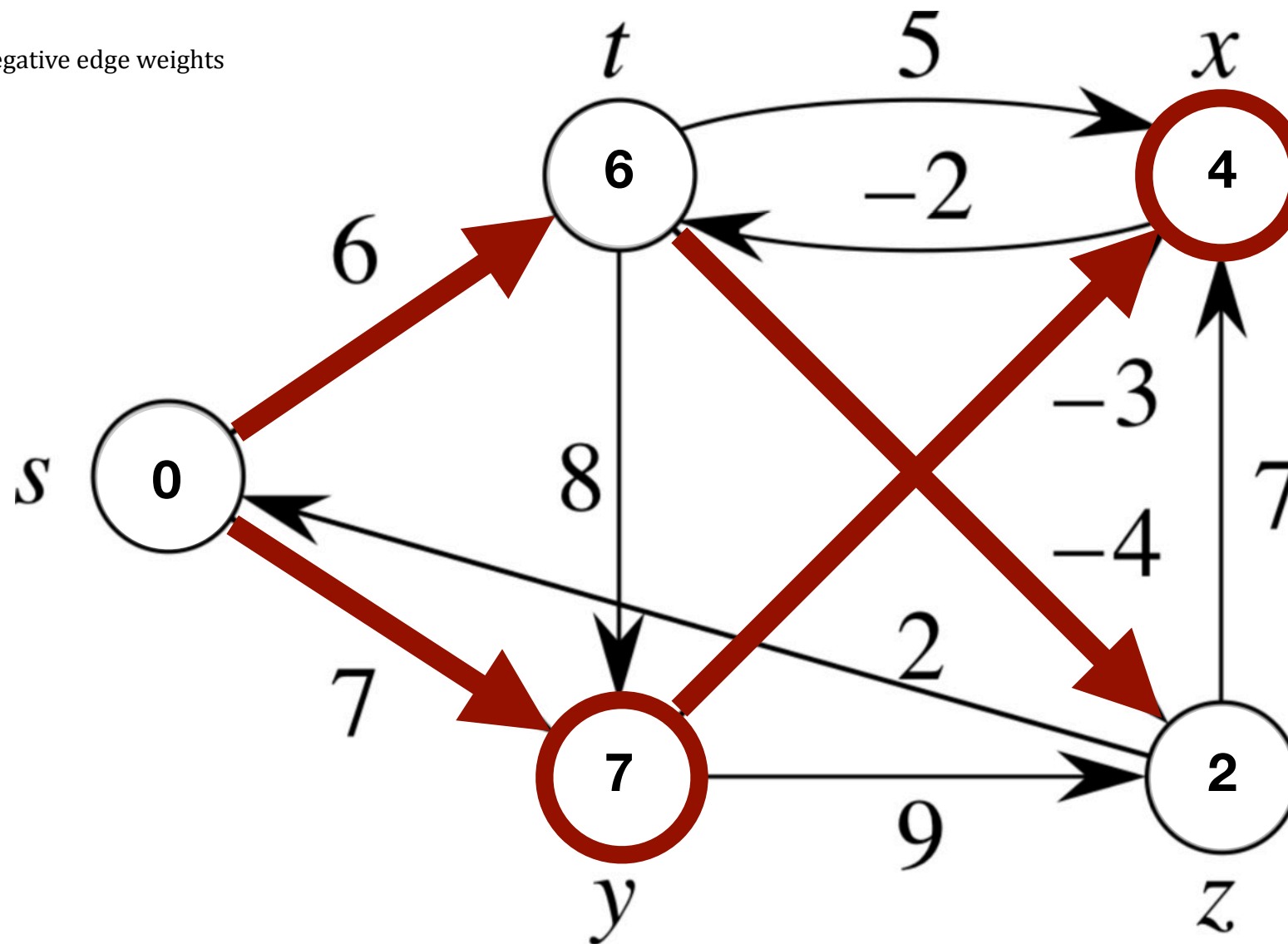
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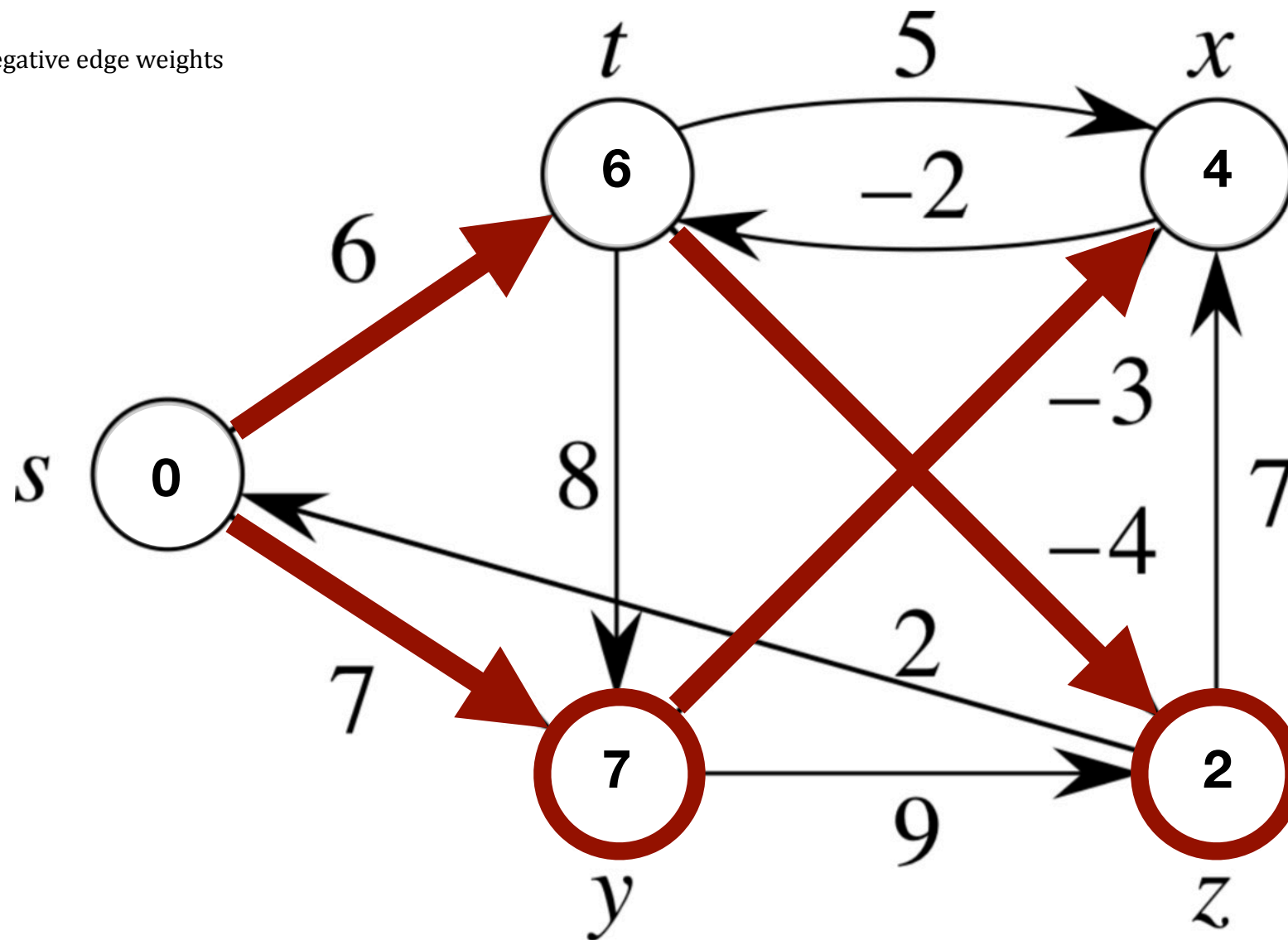
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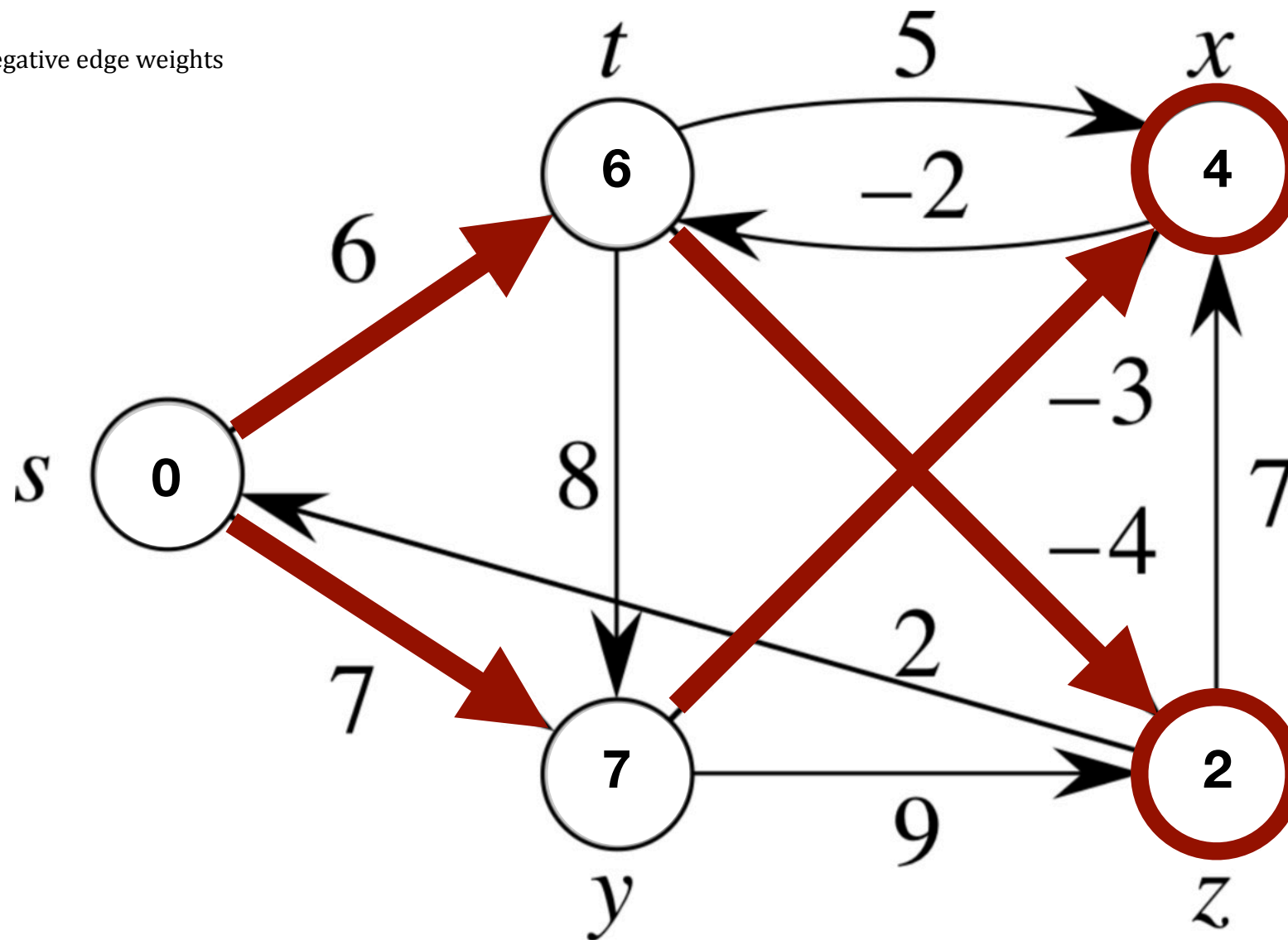
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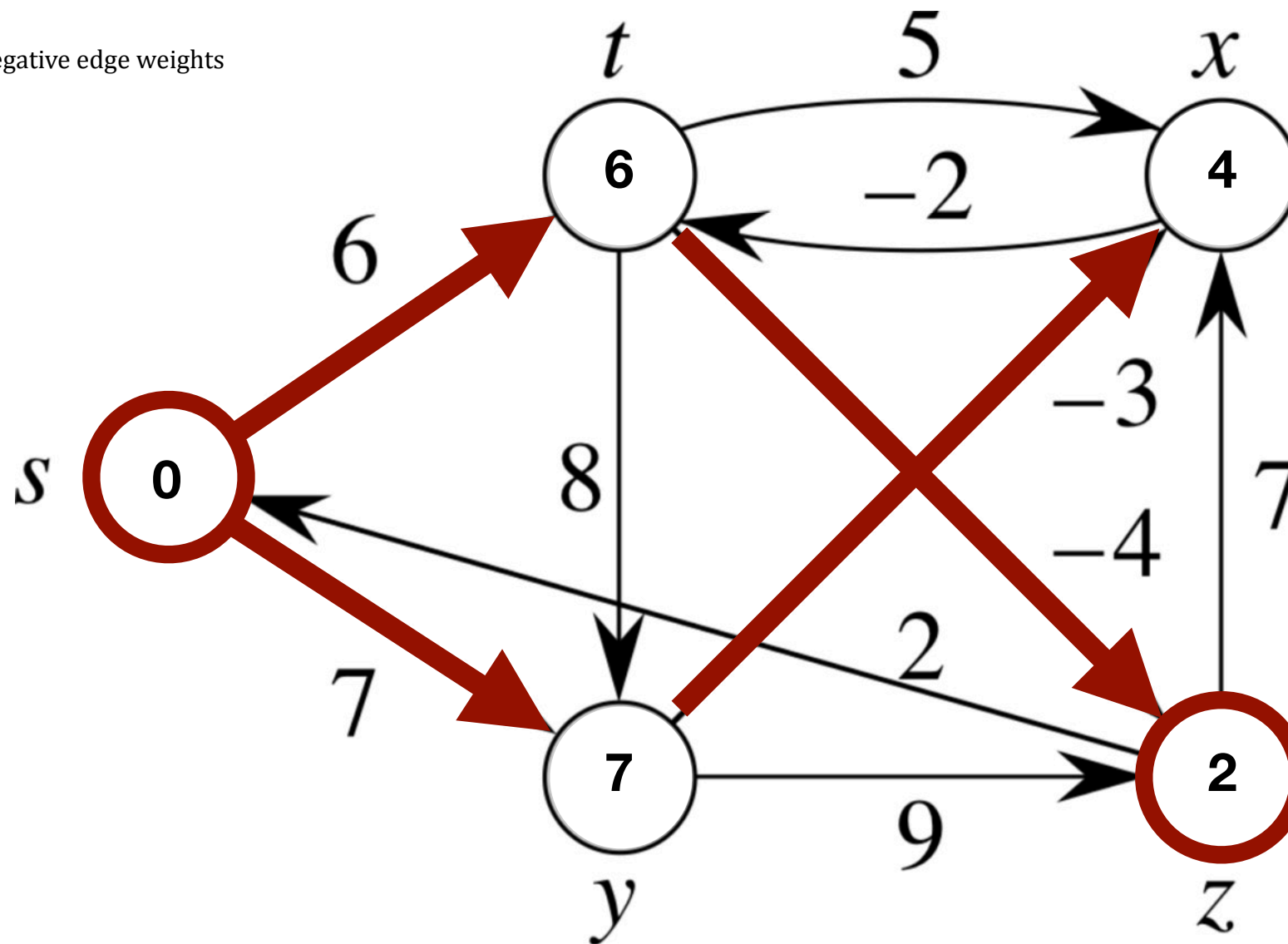
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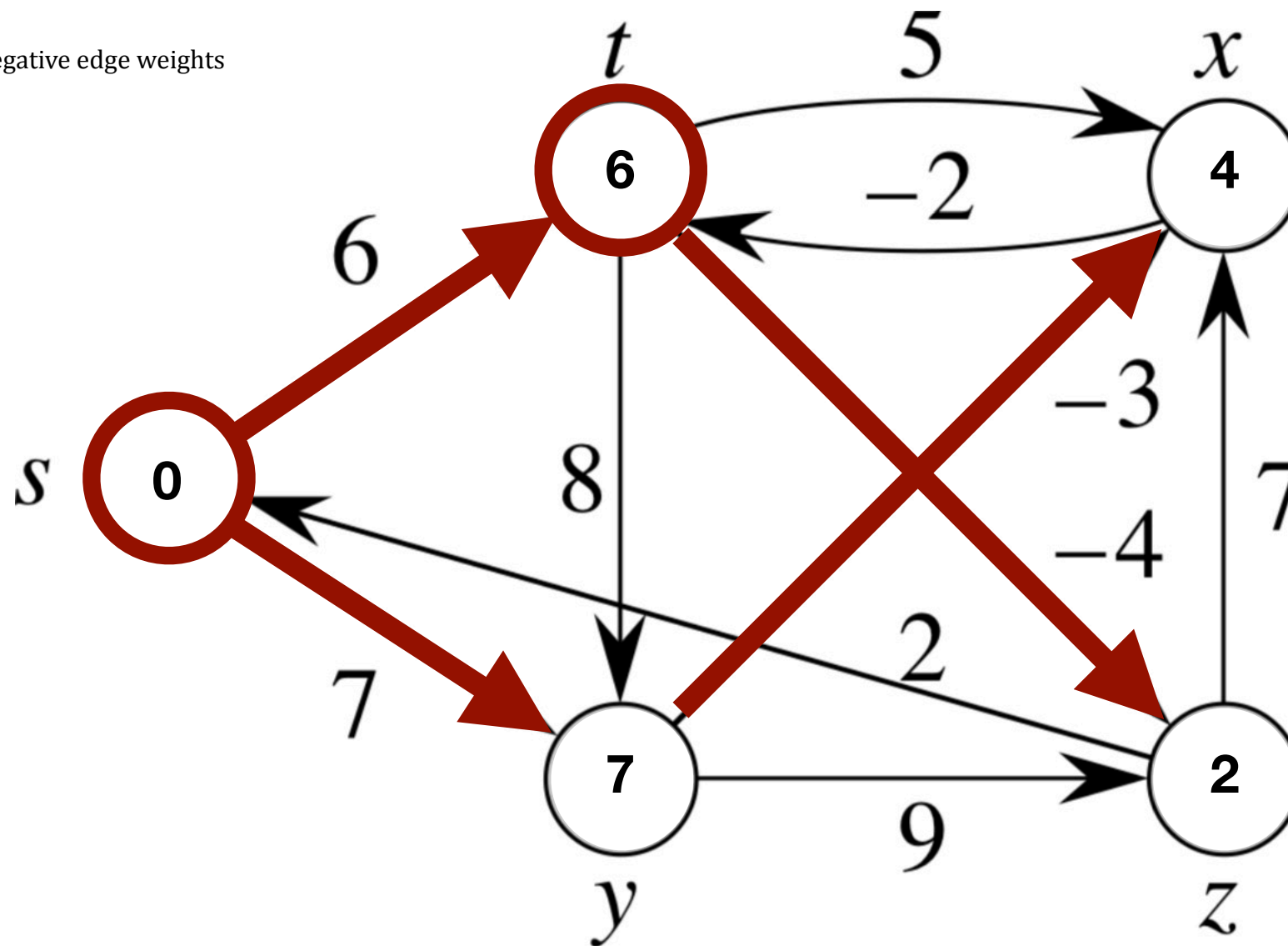
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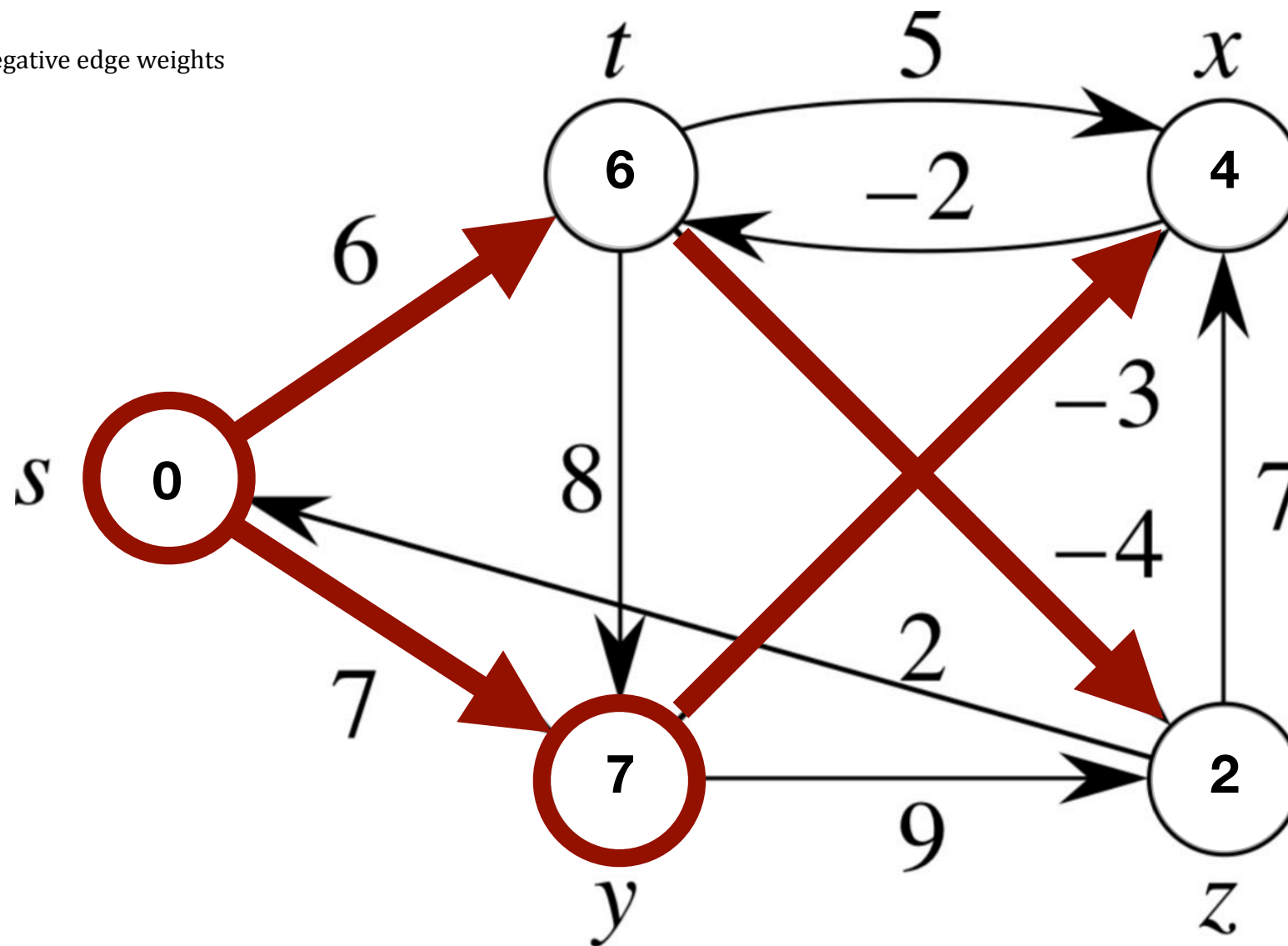
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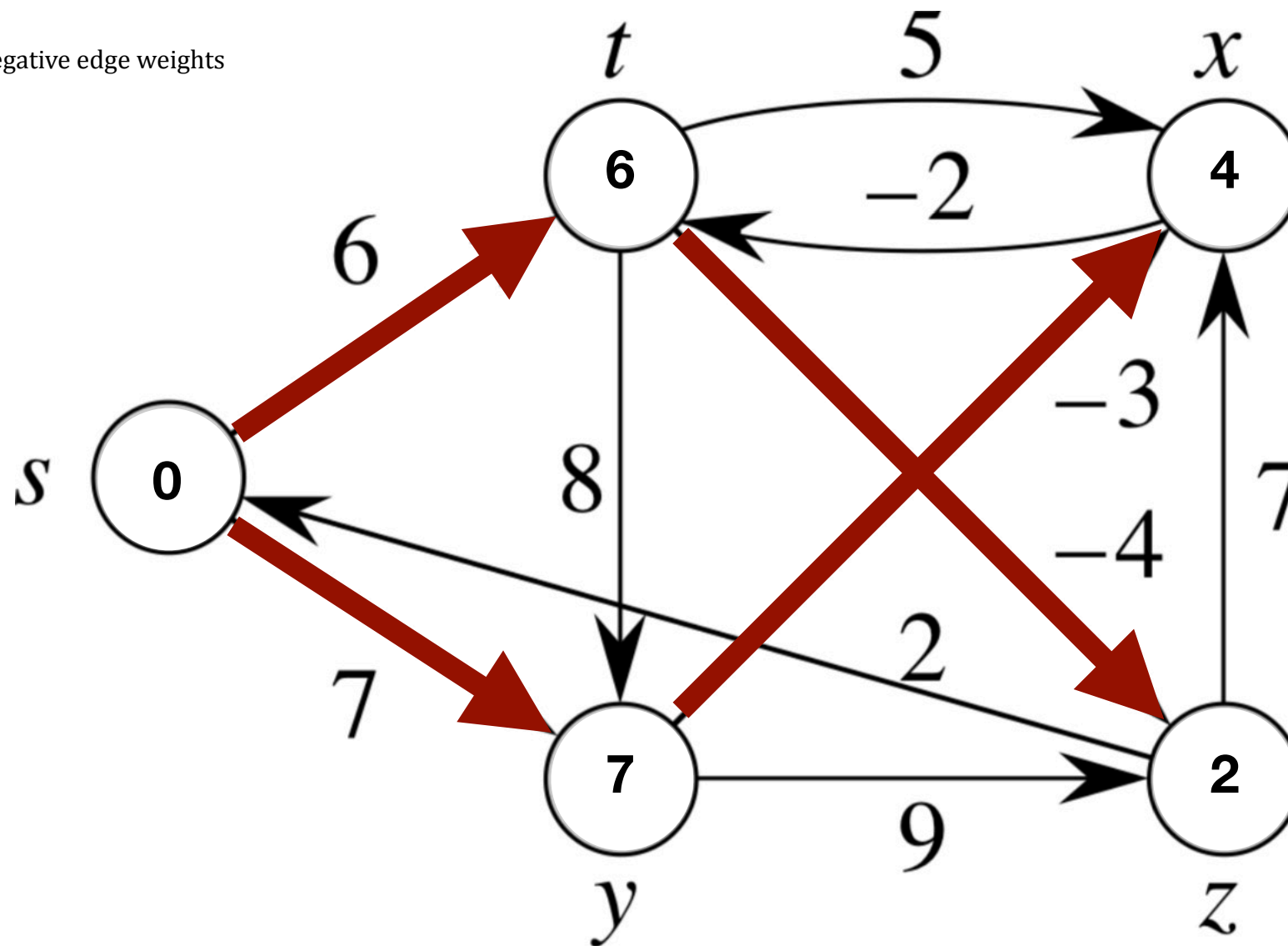
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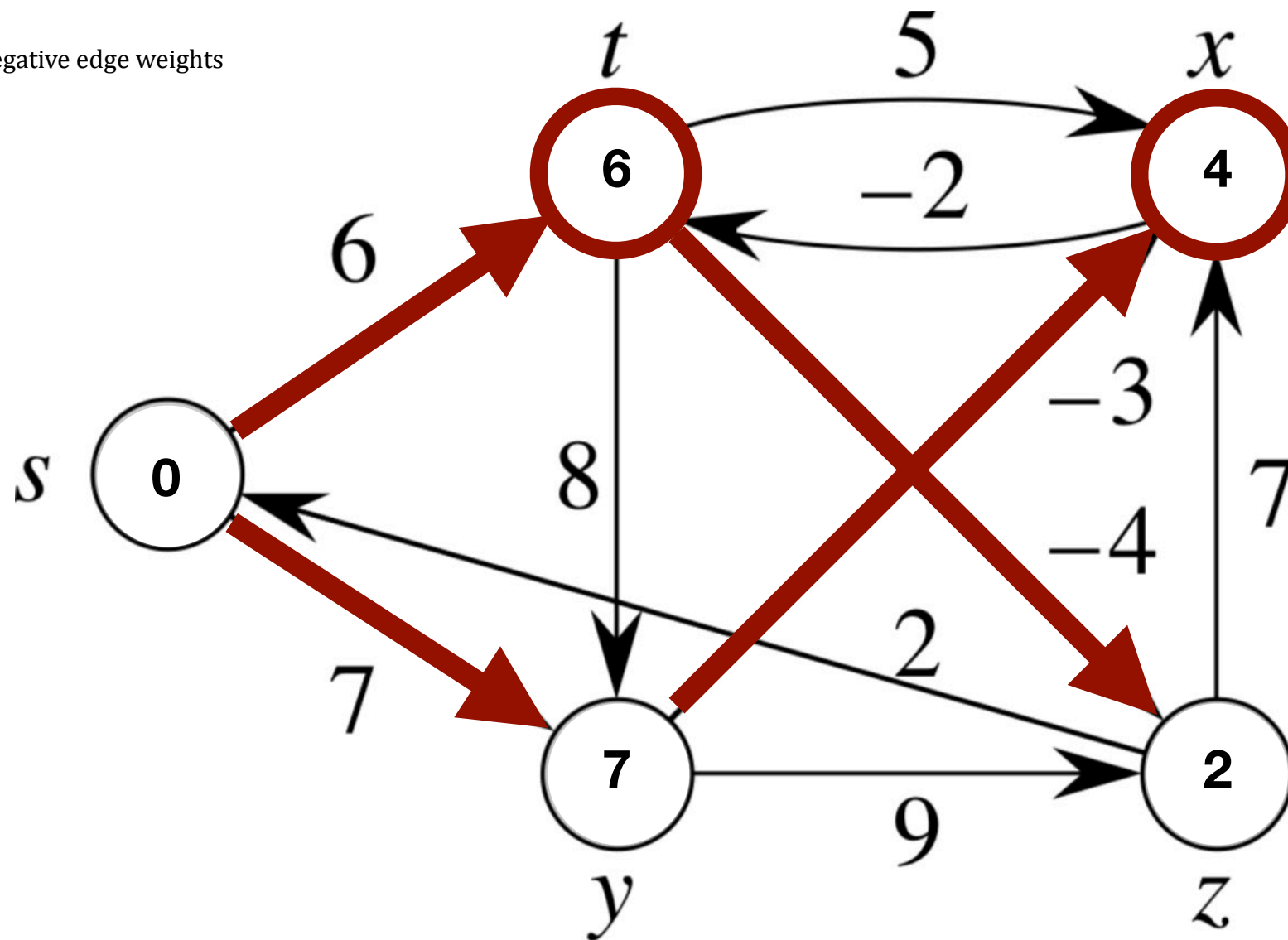
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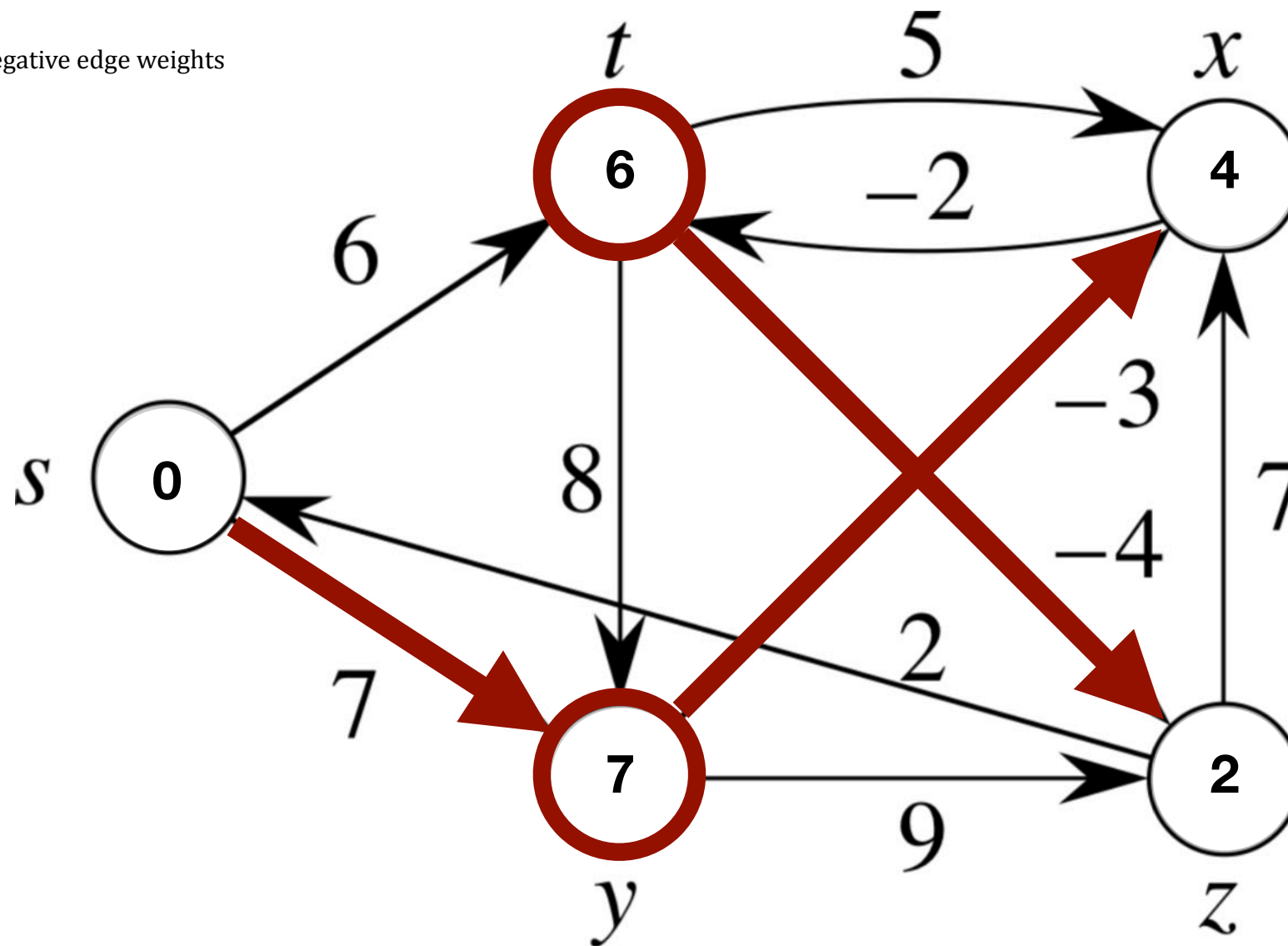
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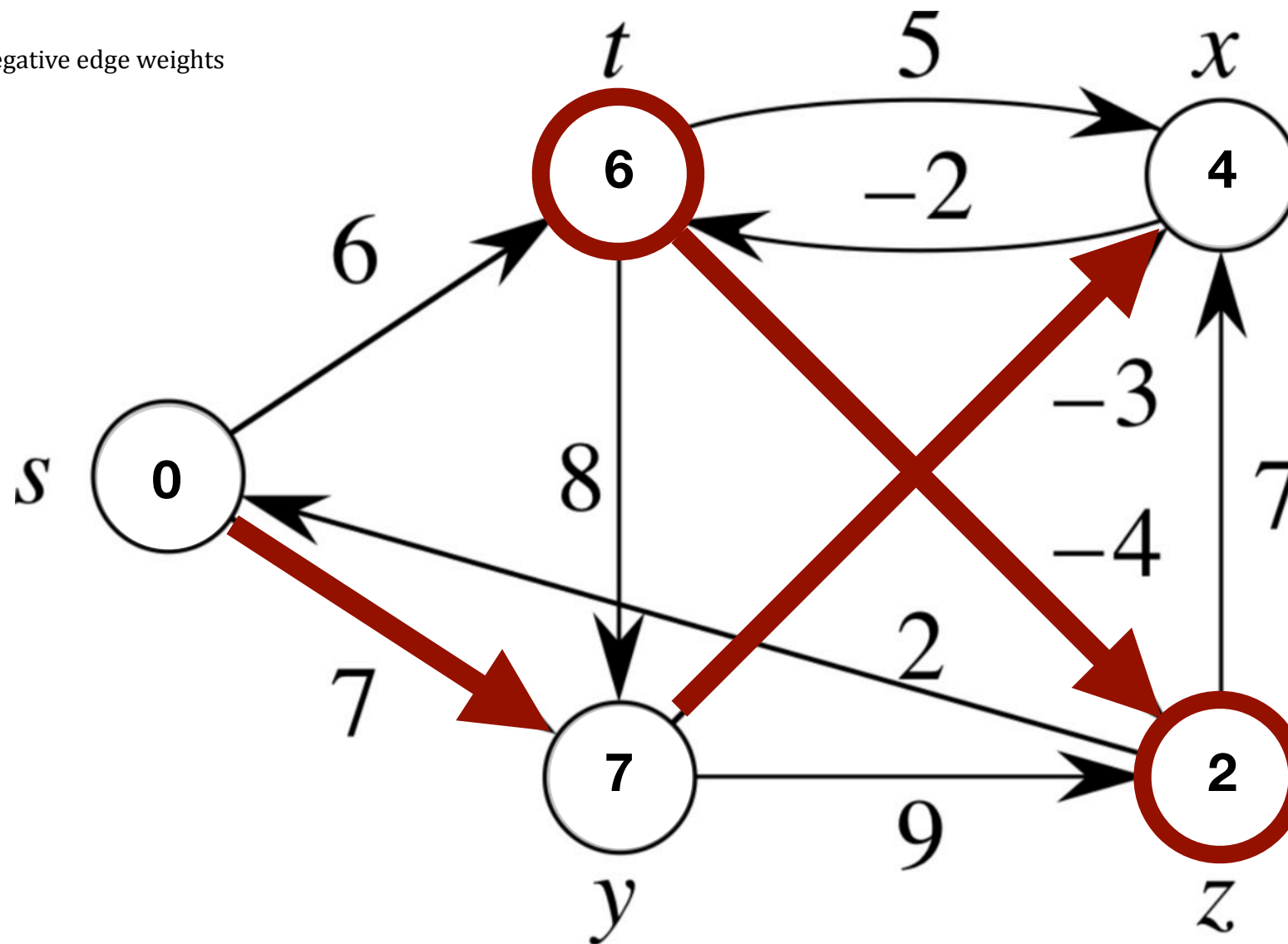
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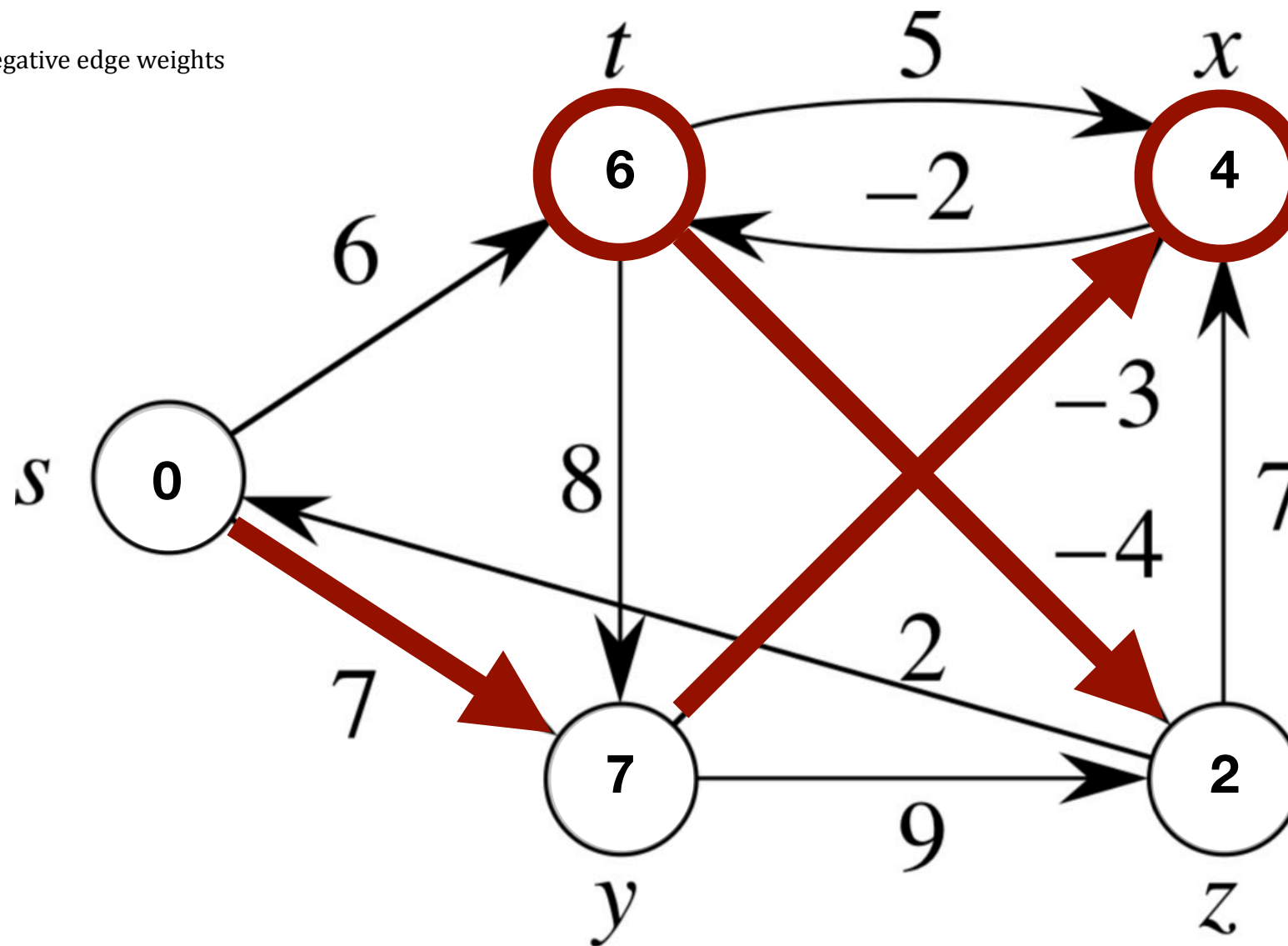
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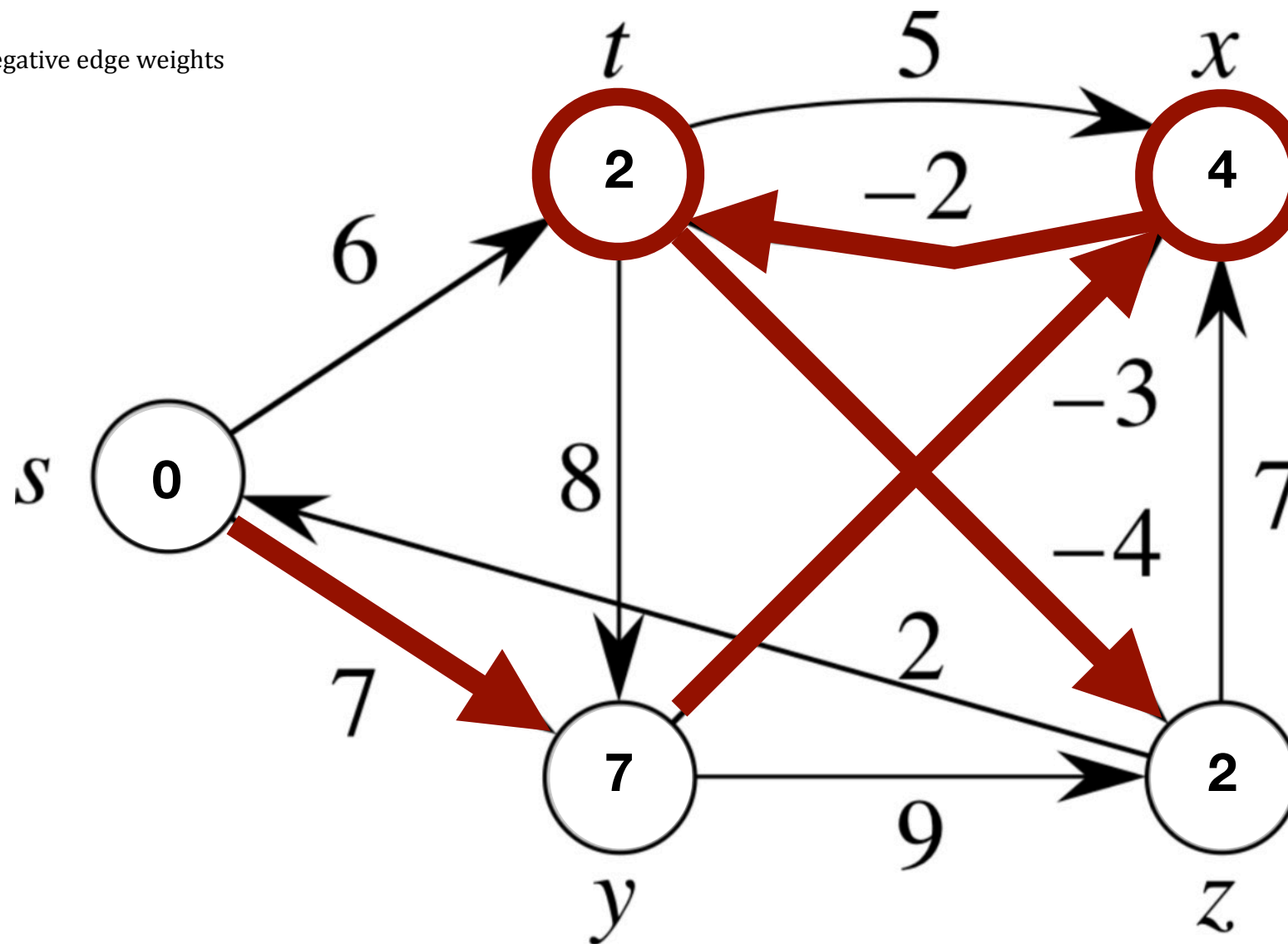
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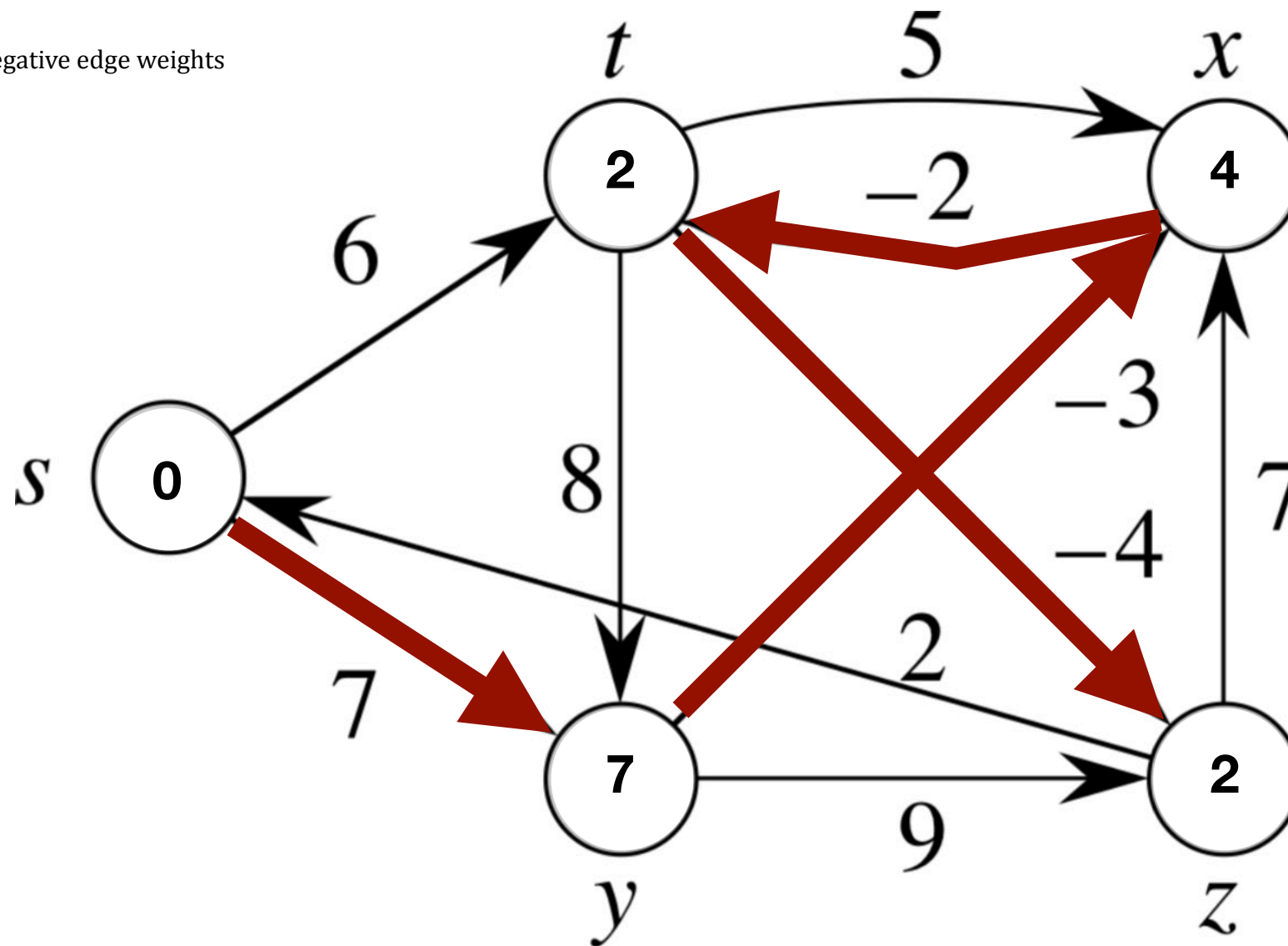
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There are no changes after this point for the rest of this pass.

But we're not yet done.

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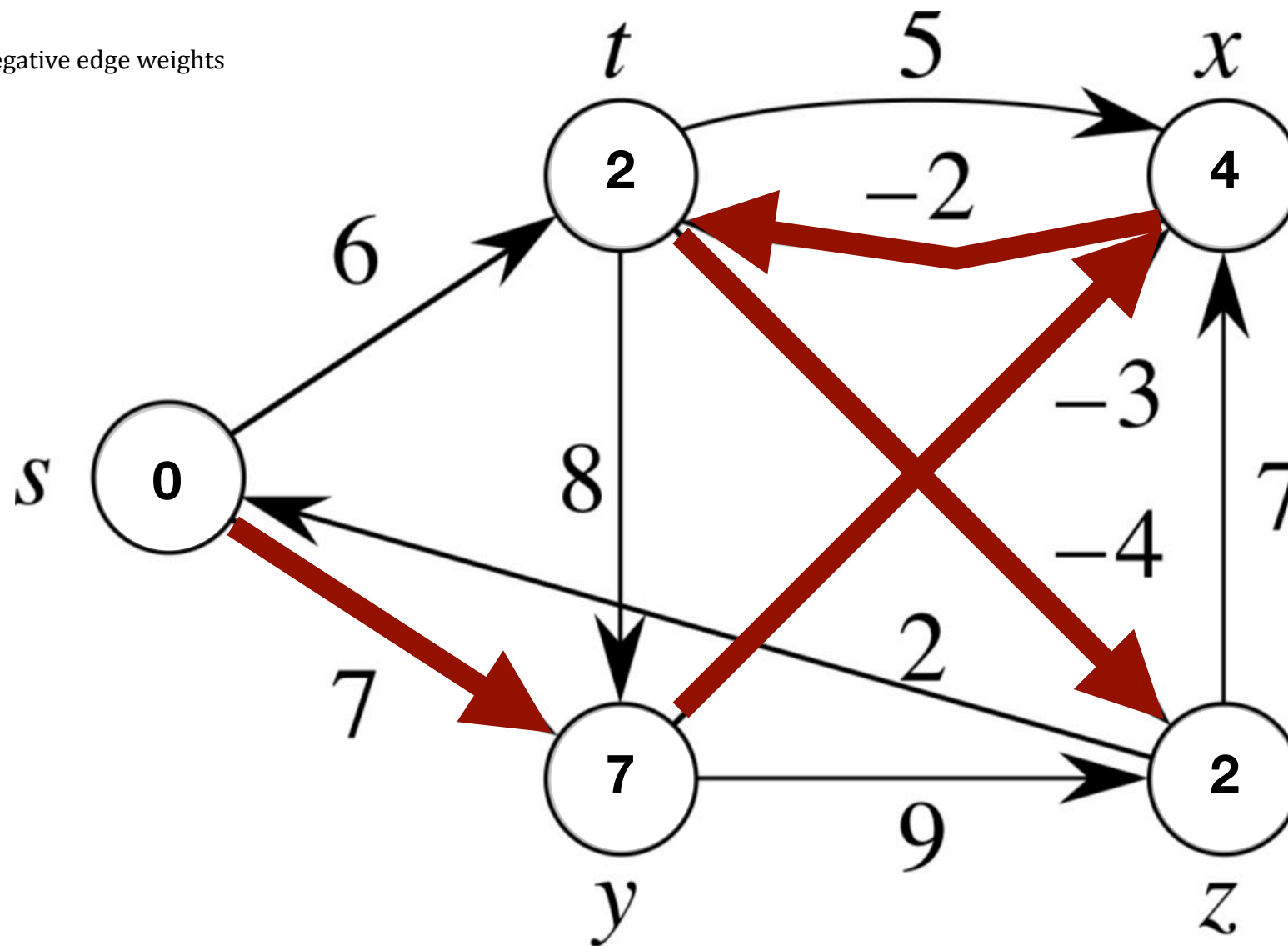
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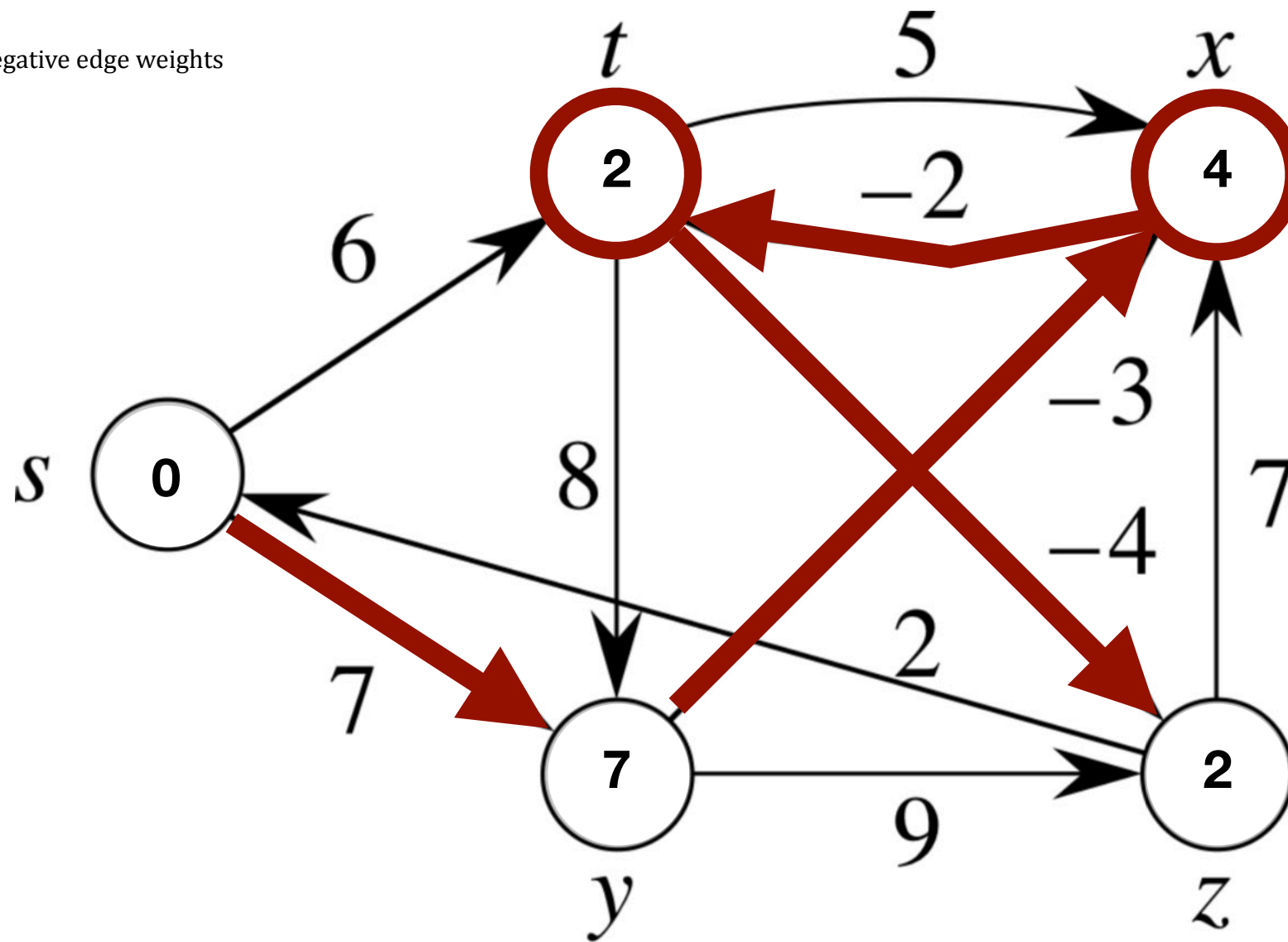
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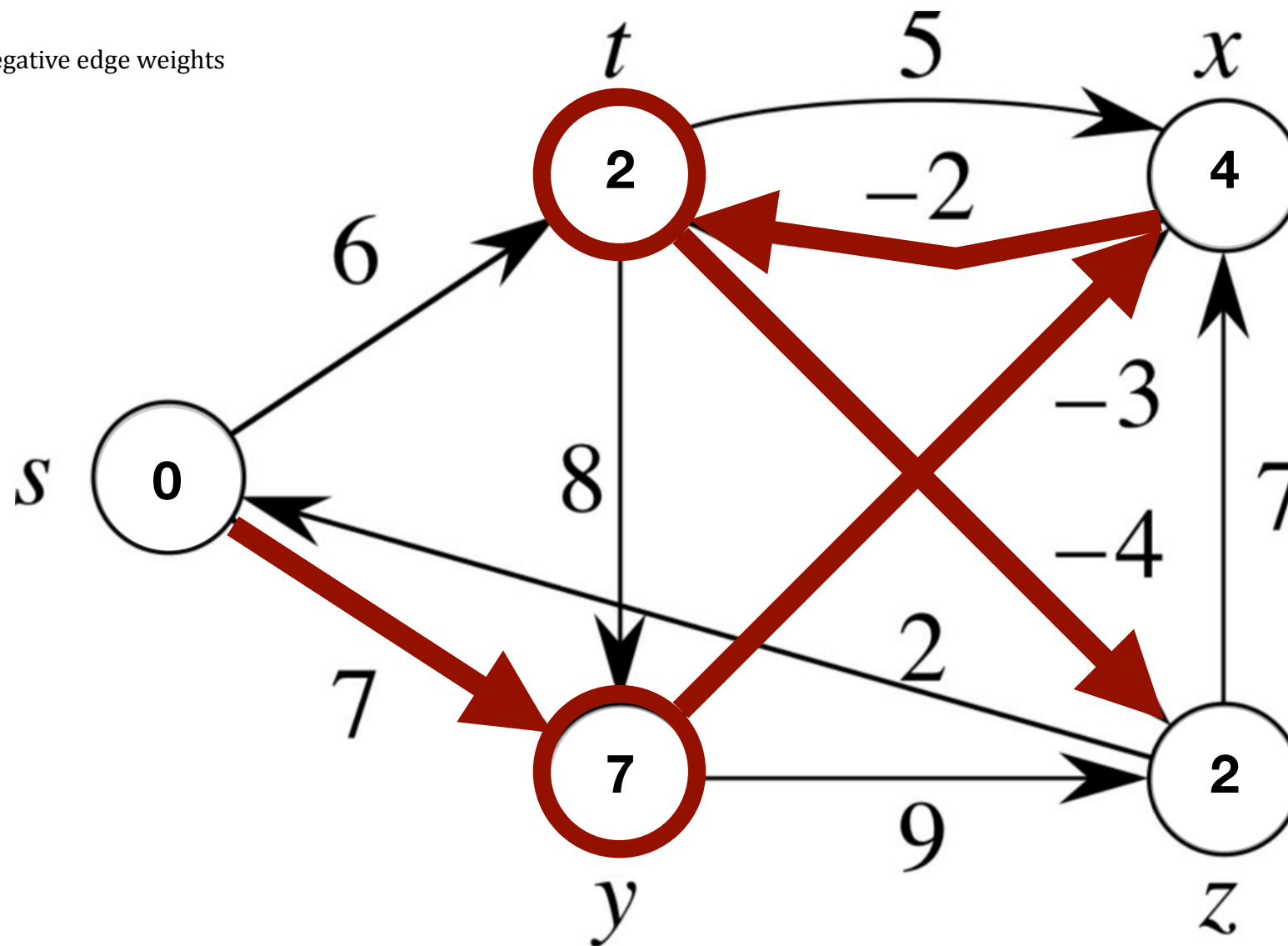
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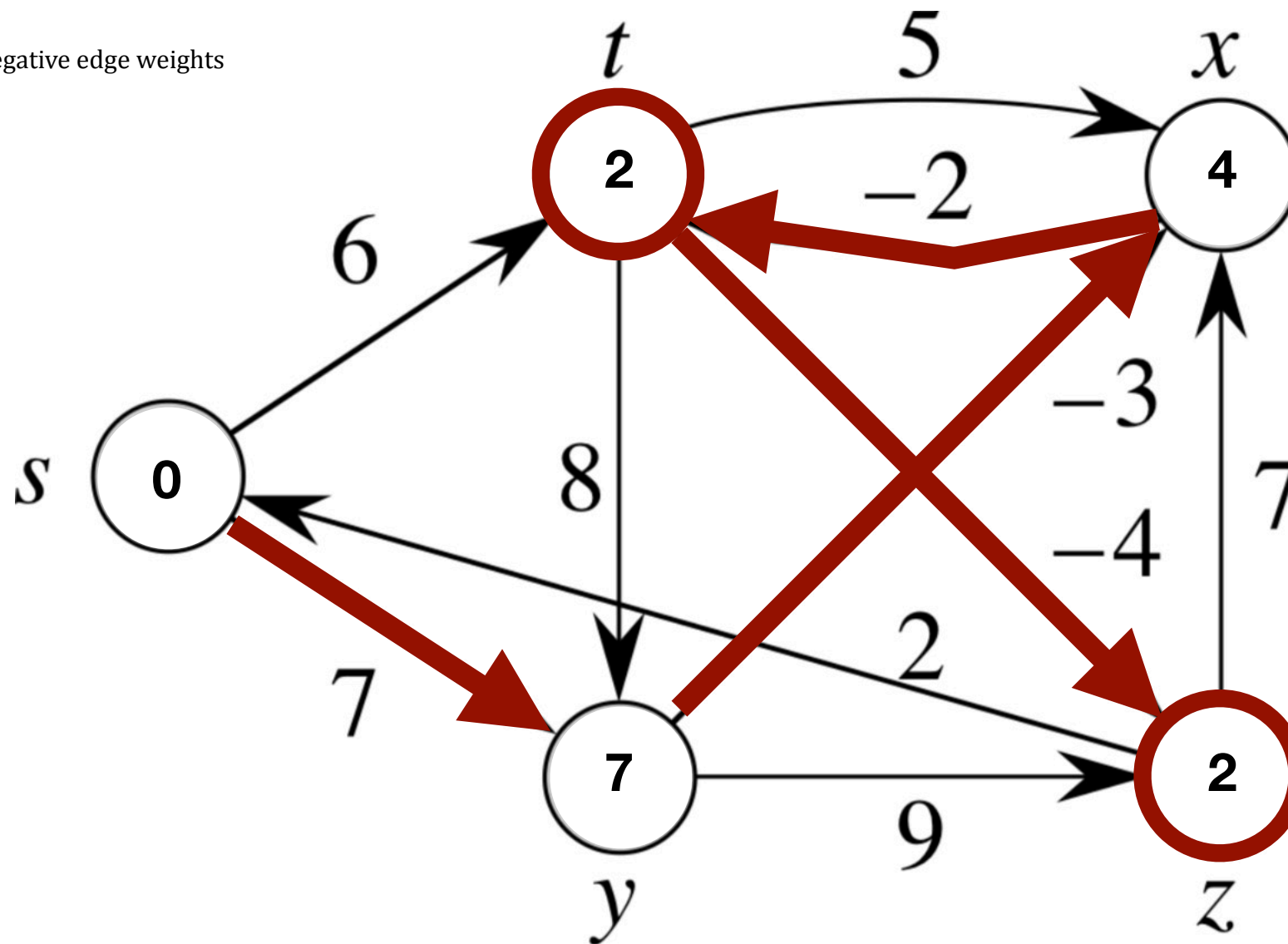
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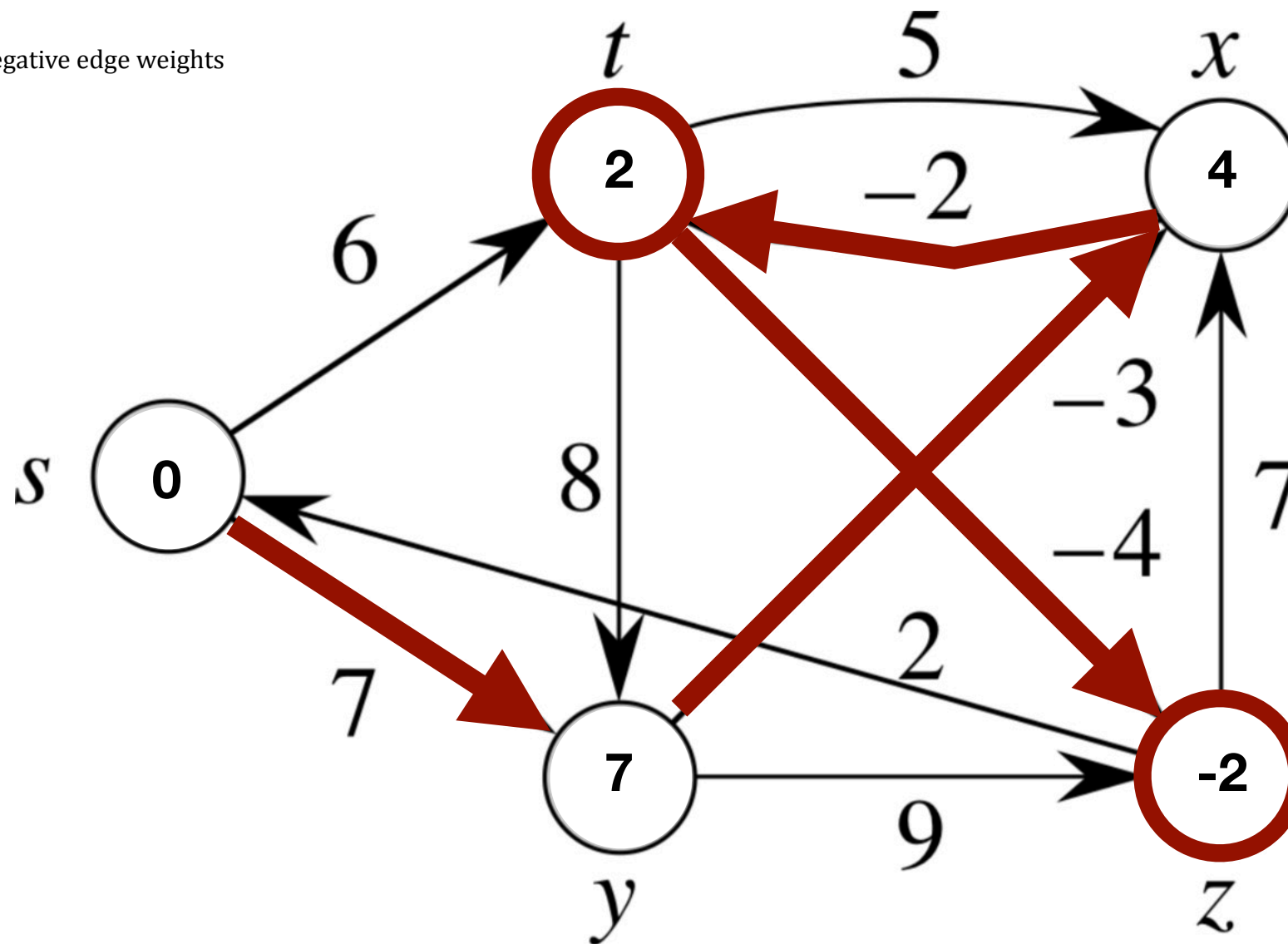
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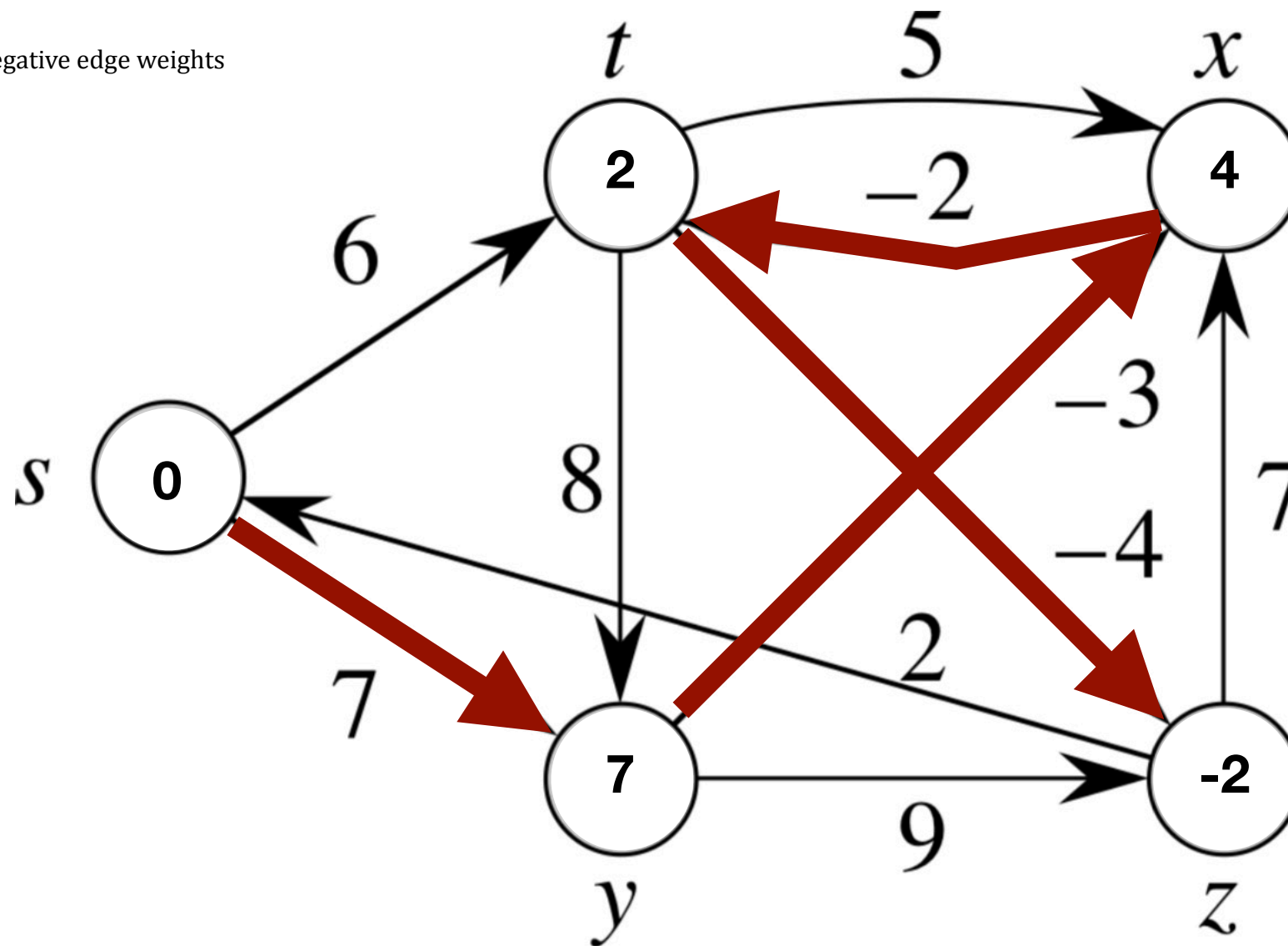
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There are no changes after this point for the rest of this pass.

And now we're done.

Almost.

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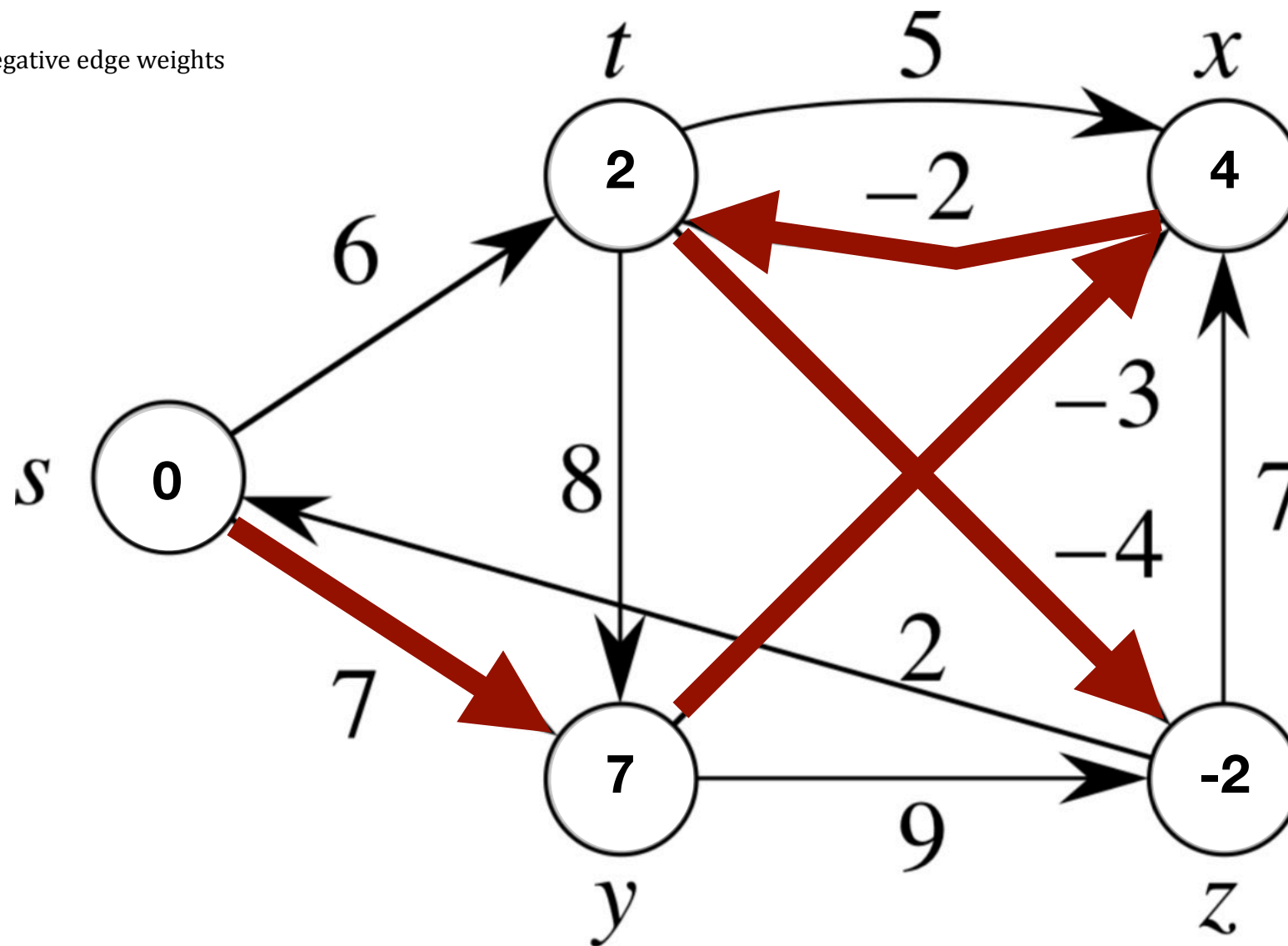
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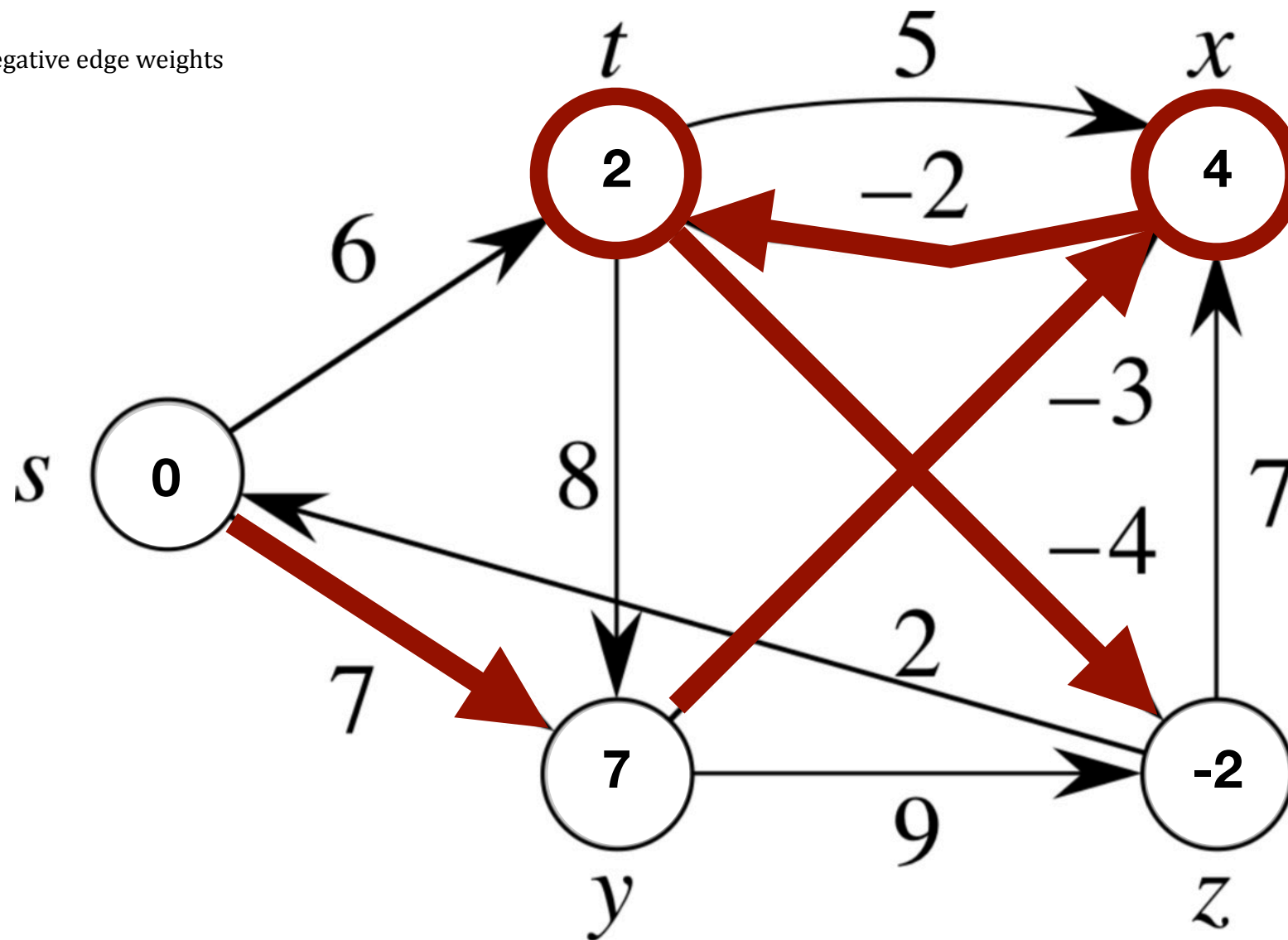
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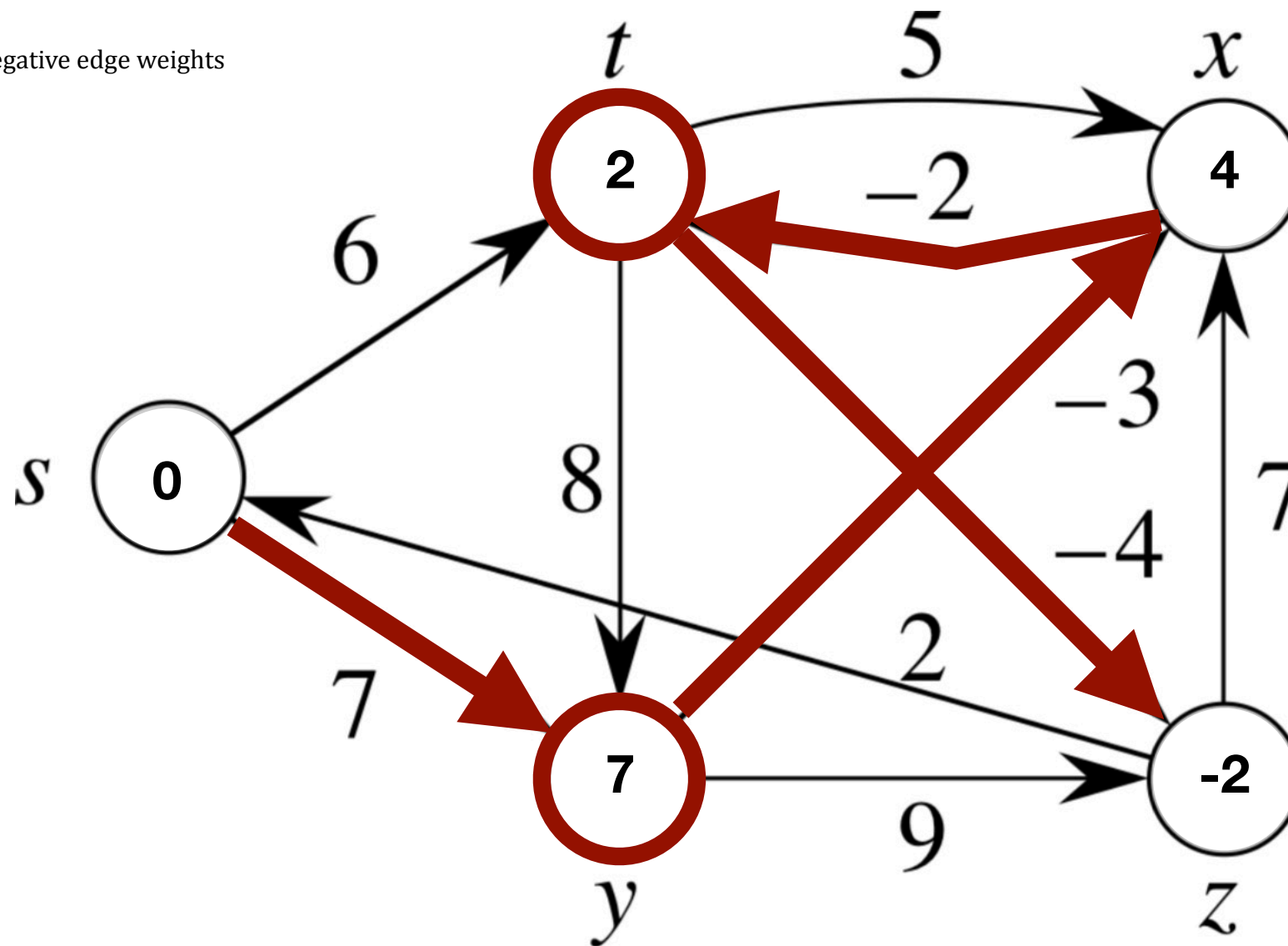
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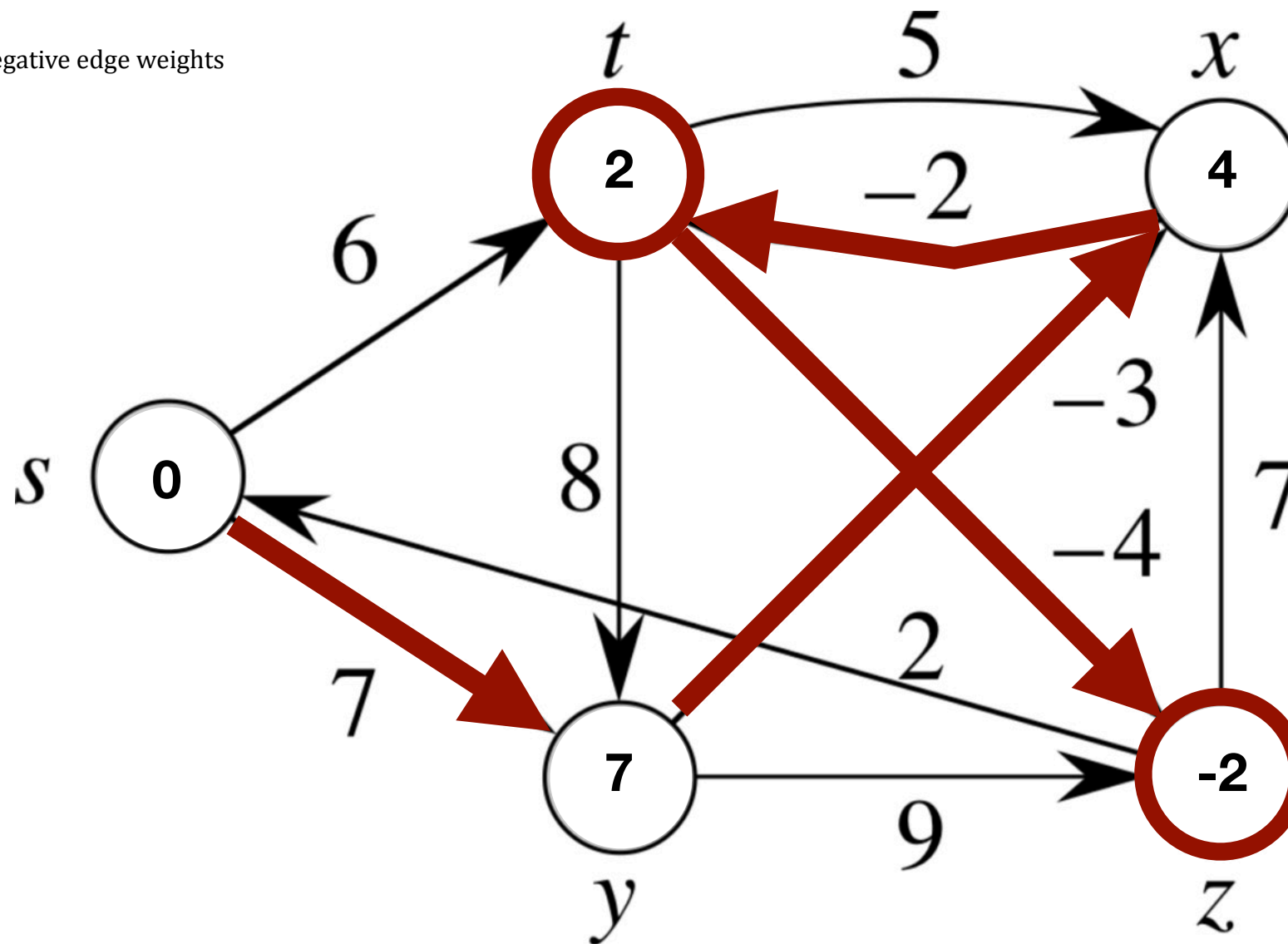
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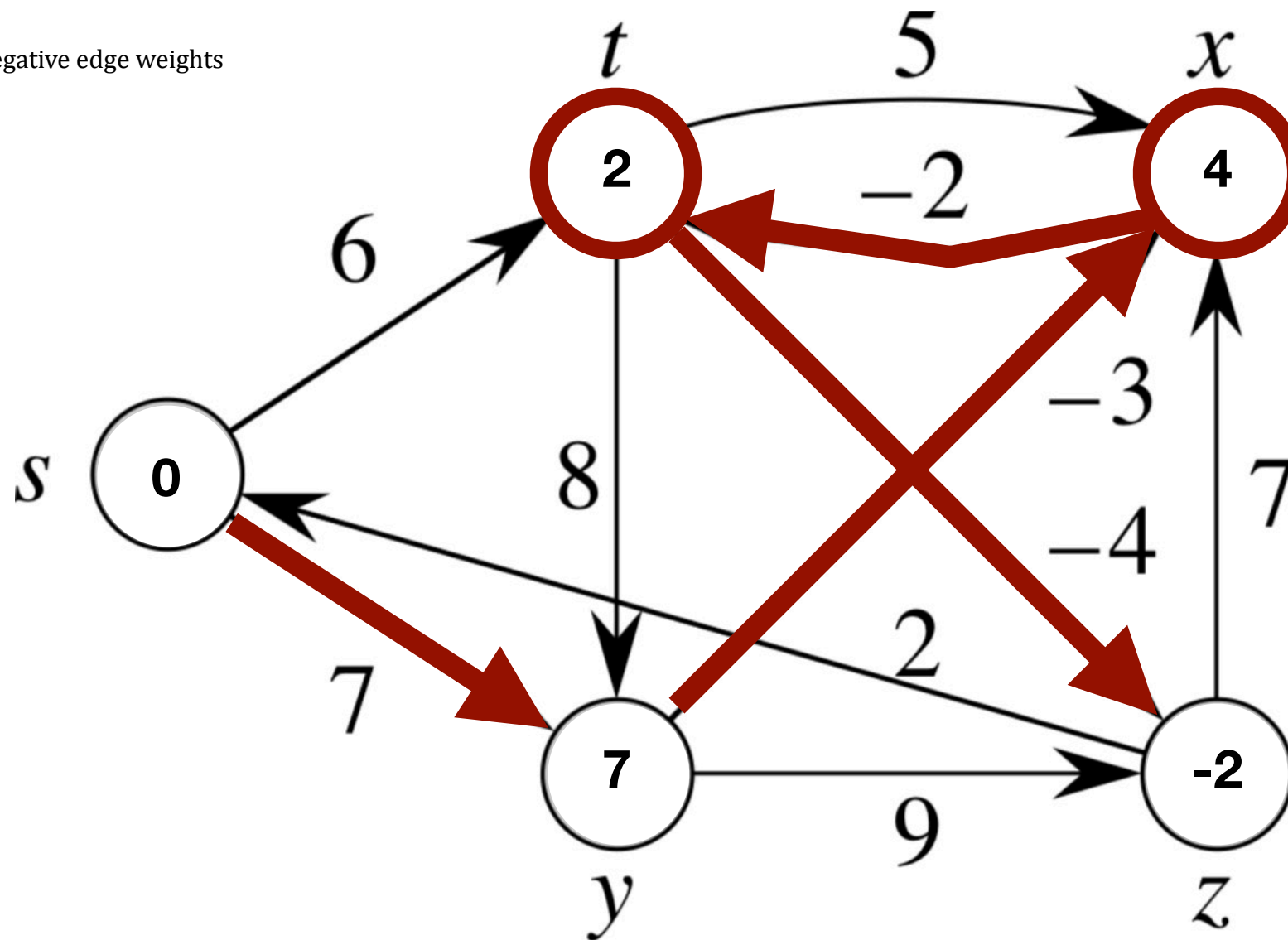
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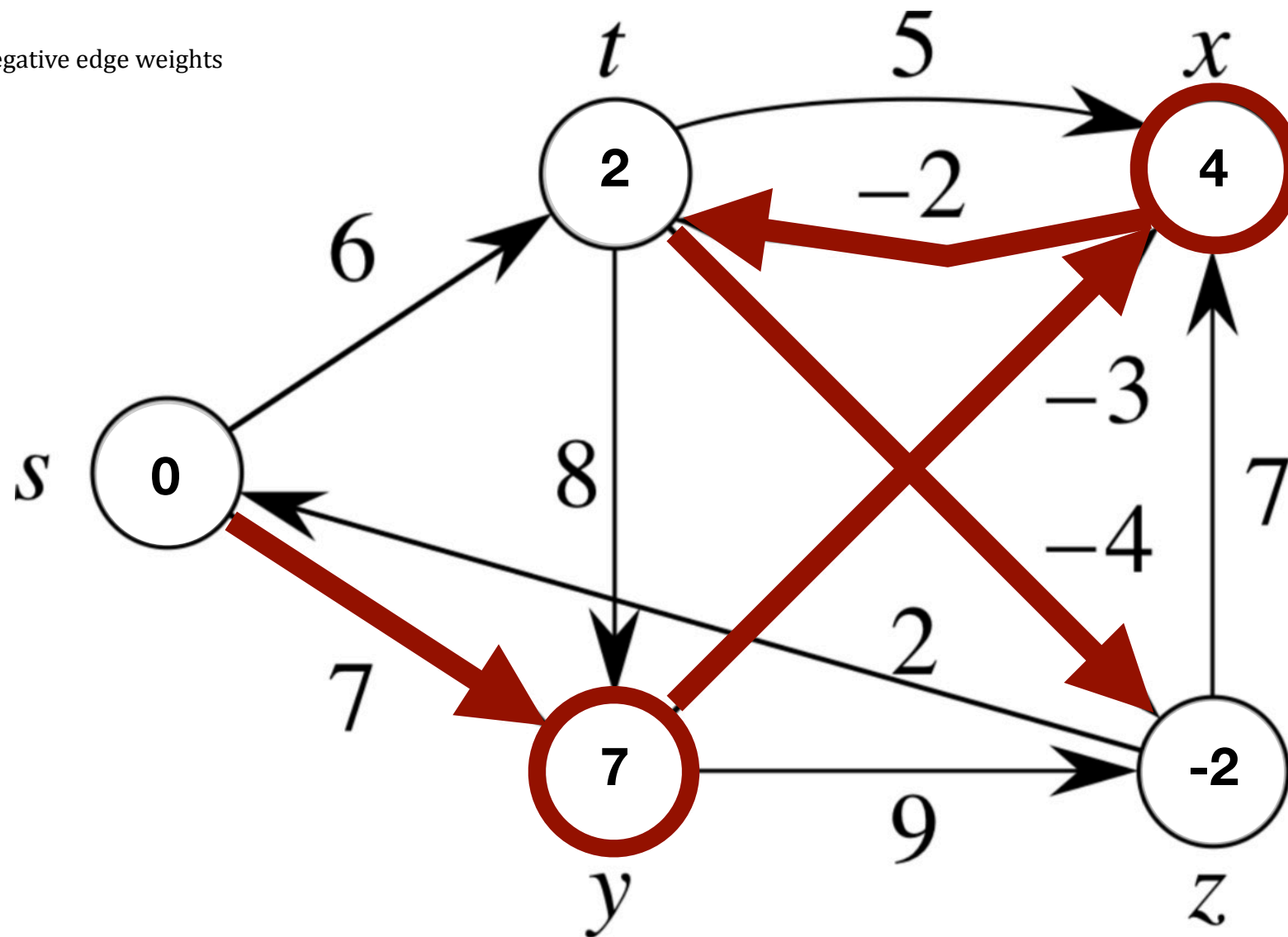
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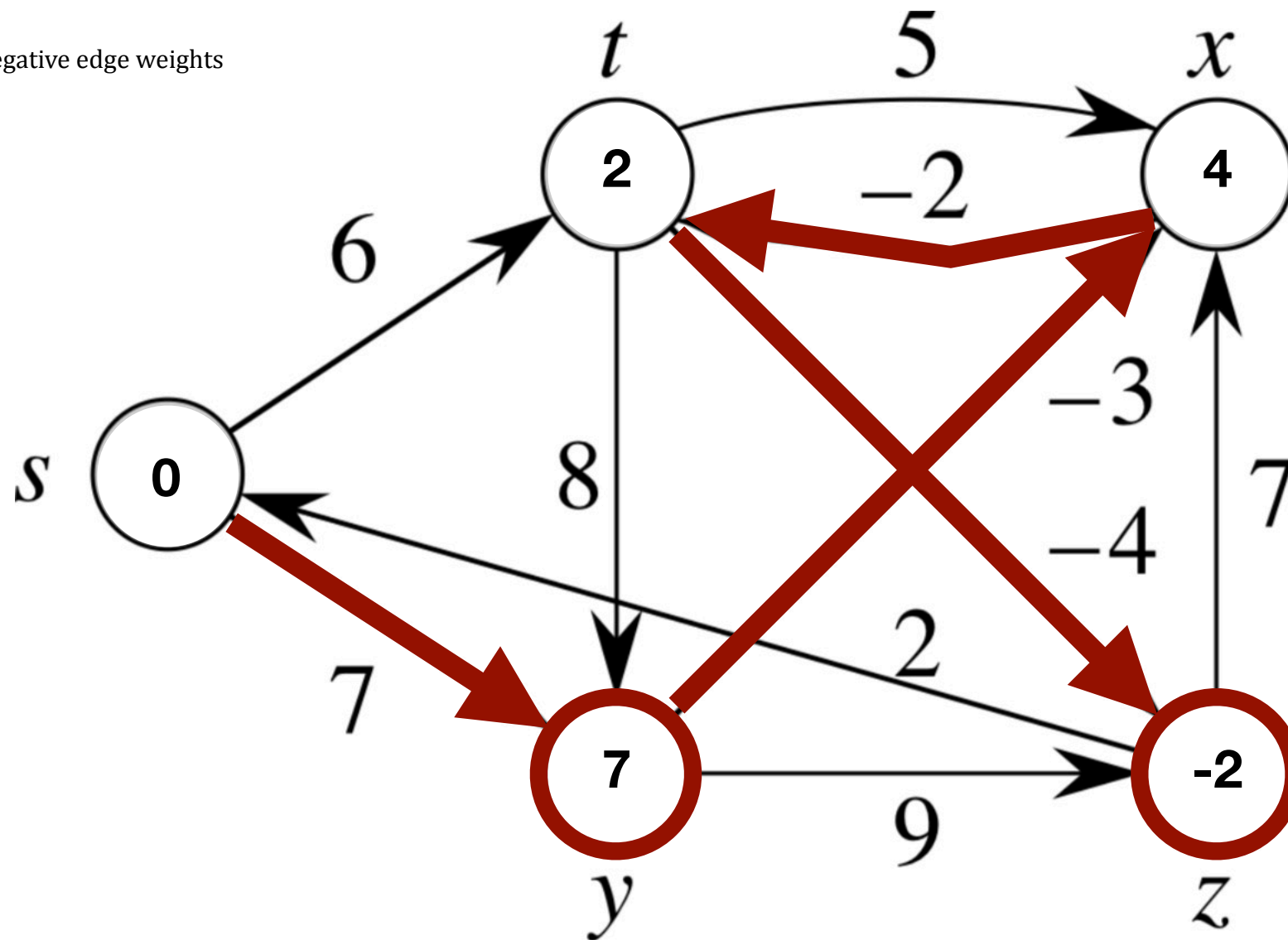
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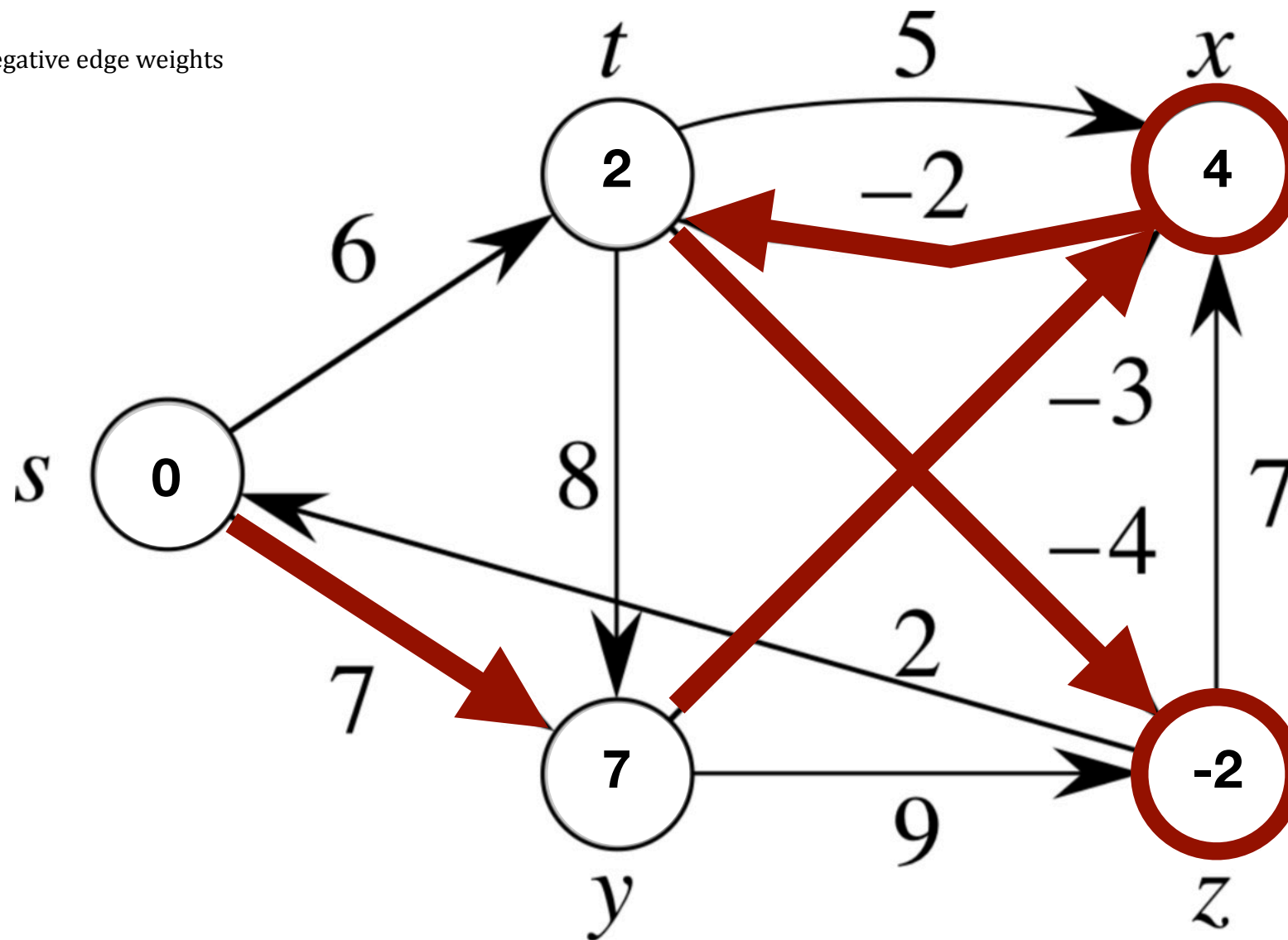
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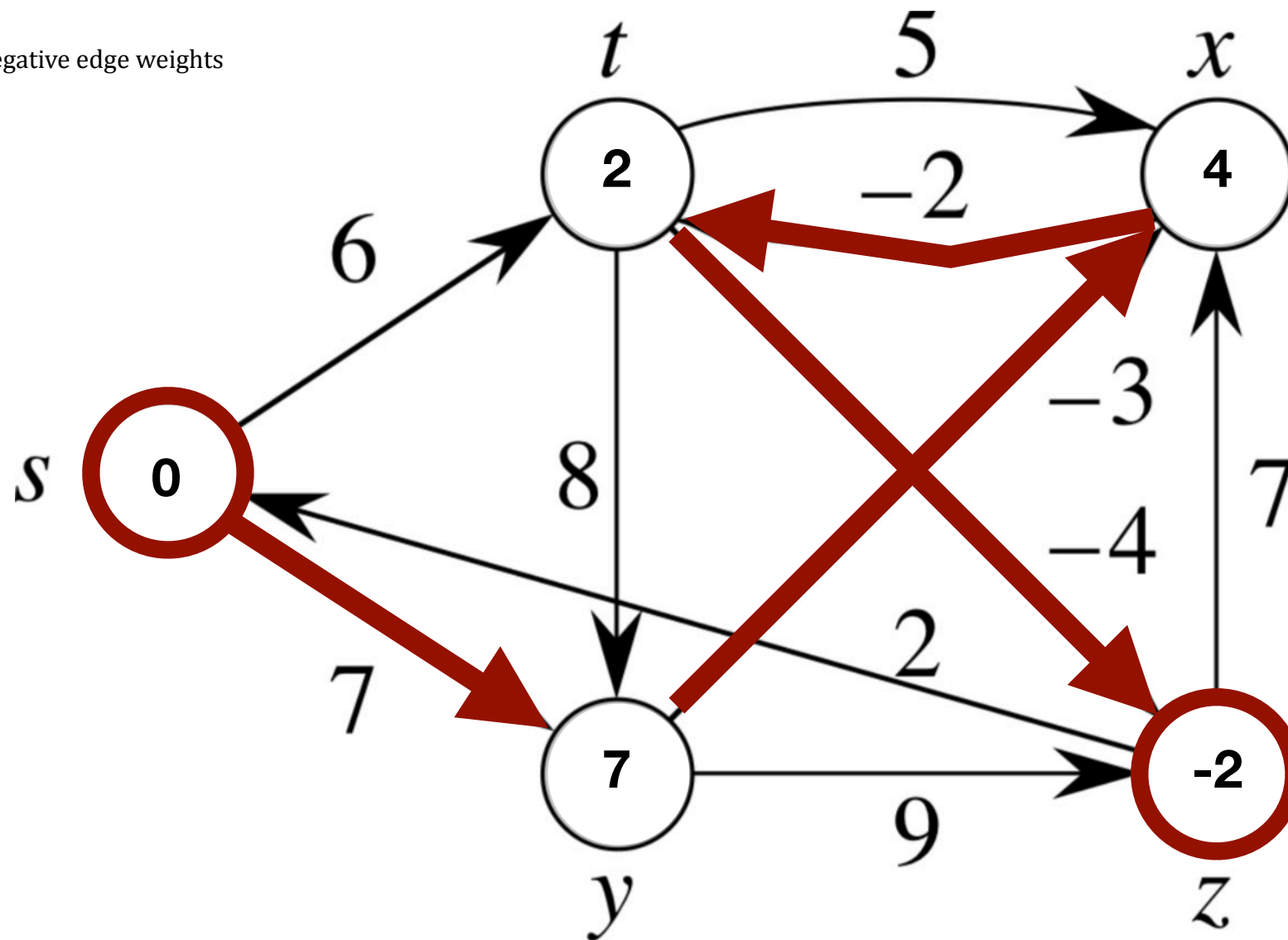
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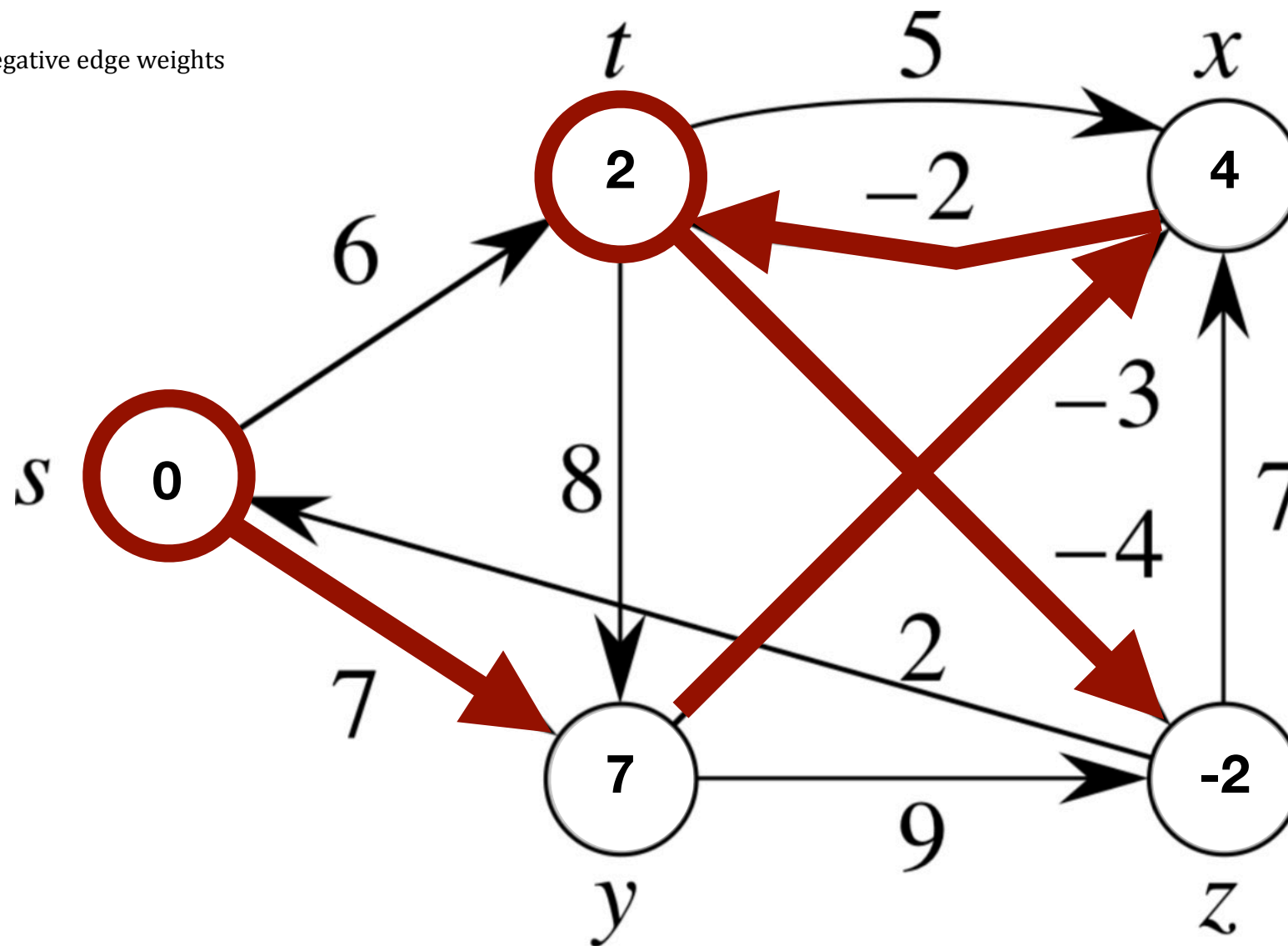
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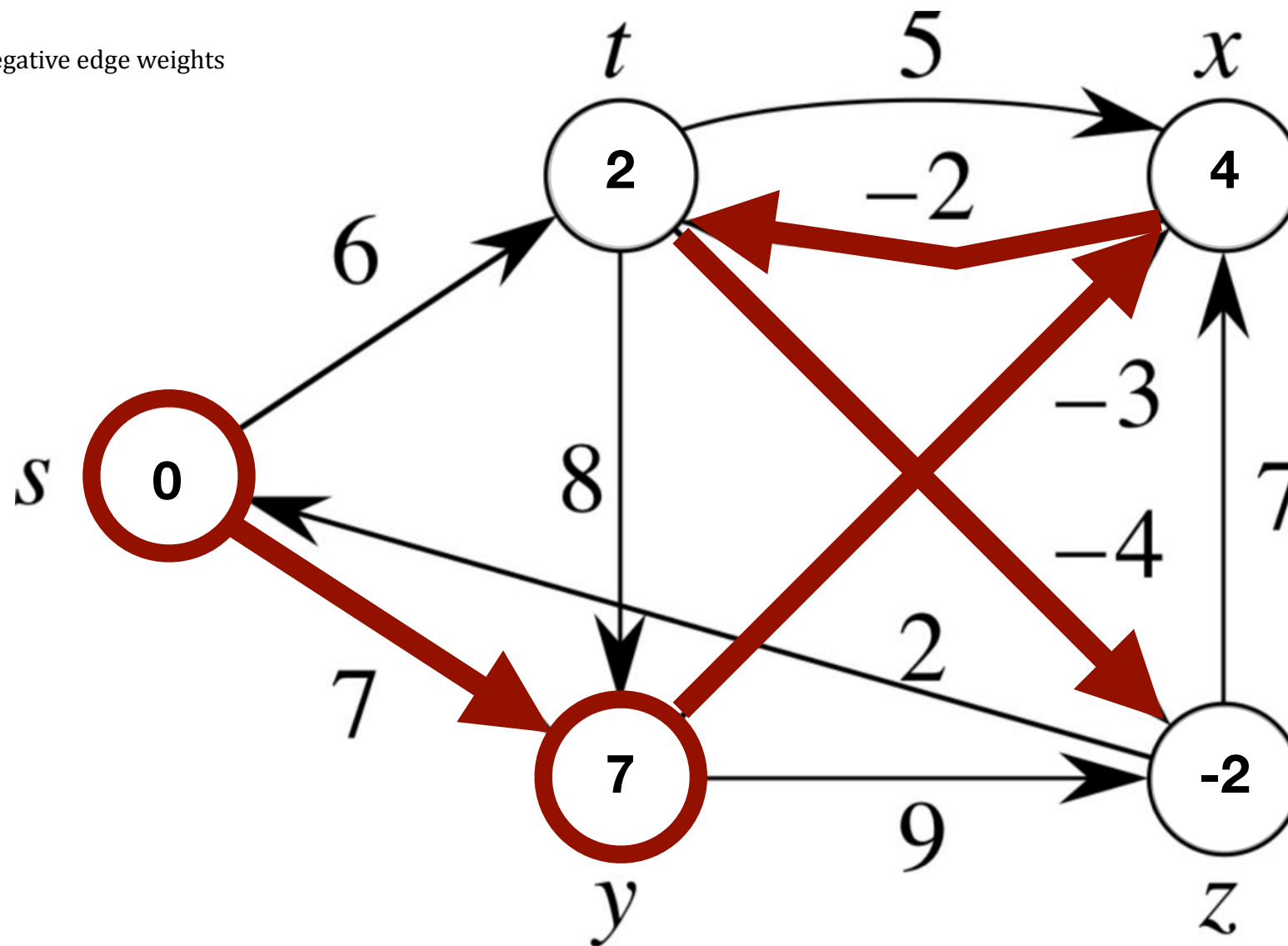
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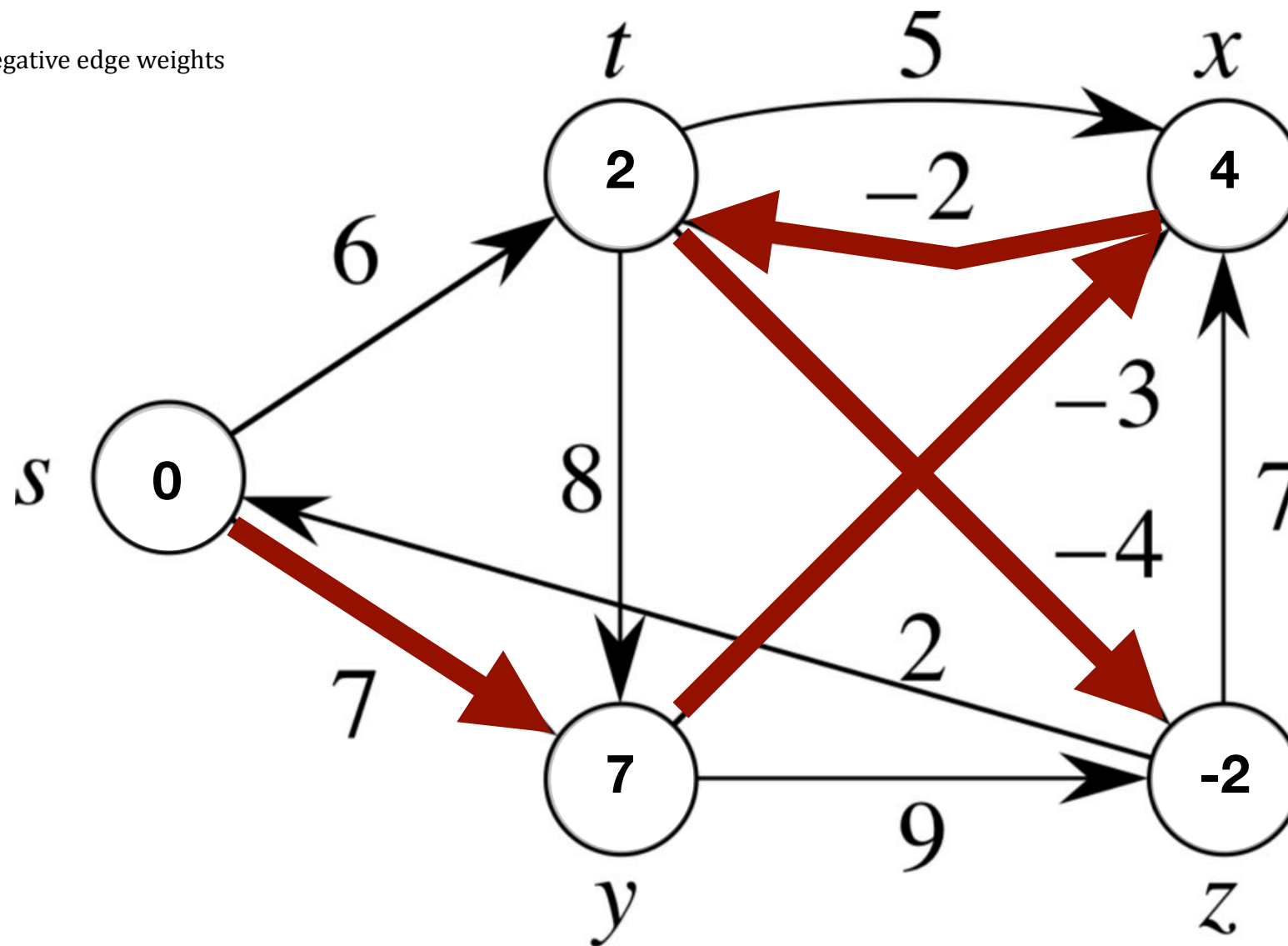
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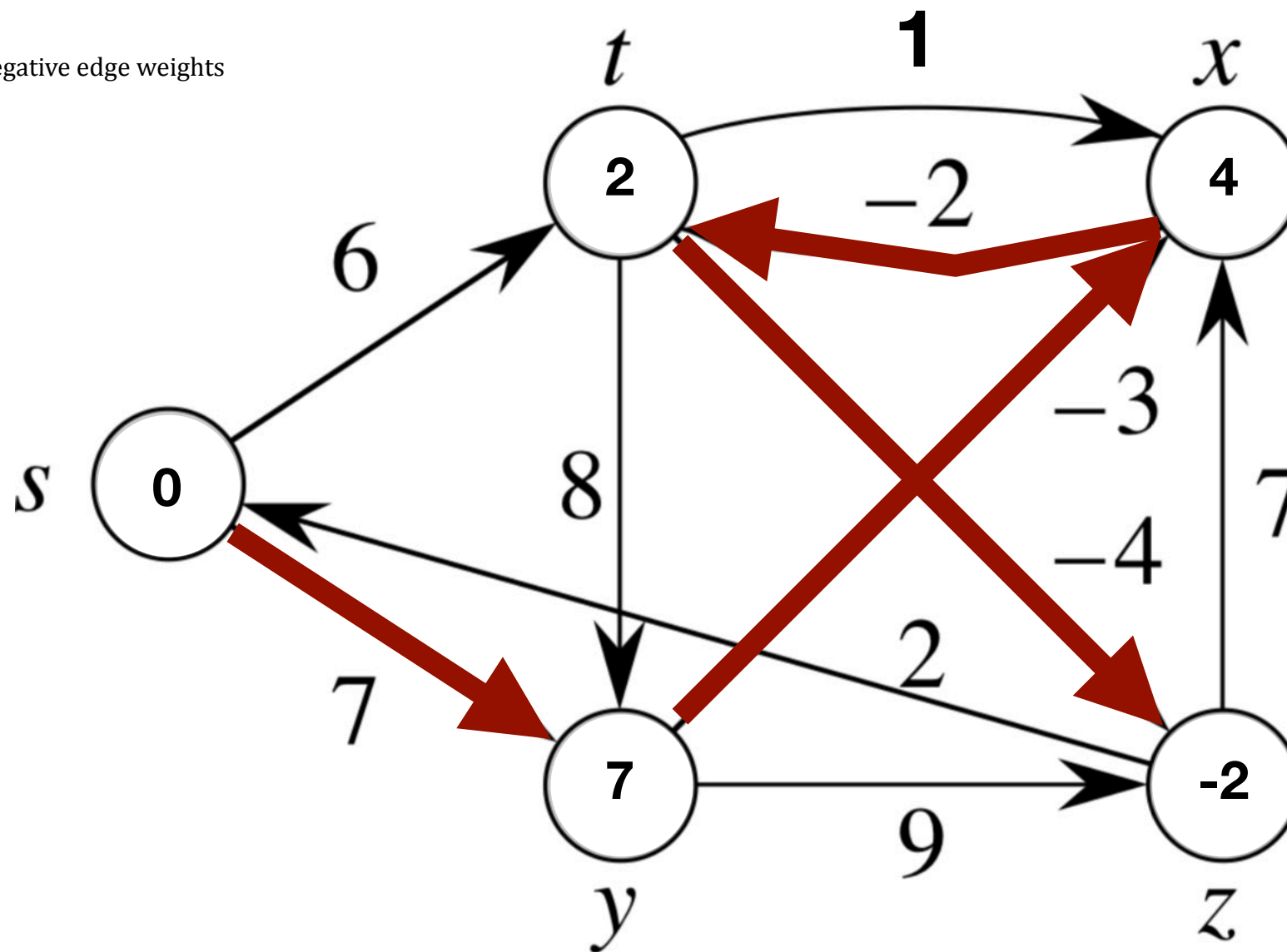
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What if $w(t,x)=1$?

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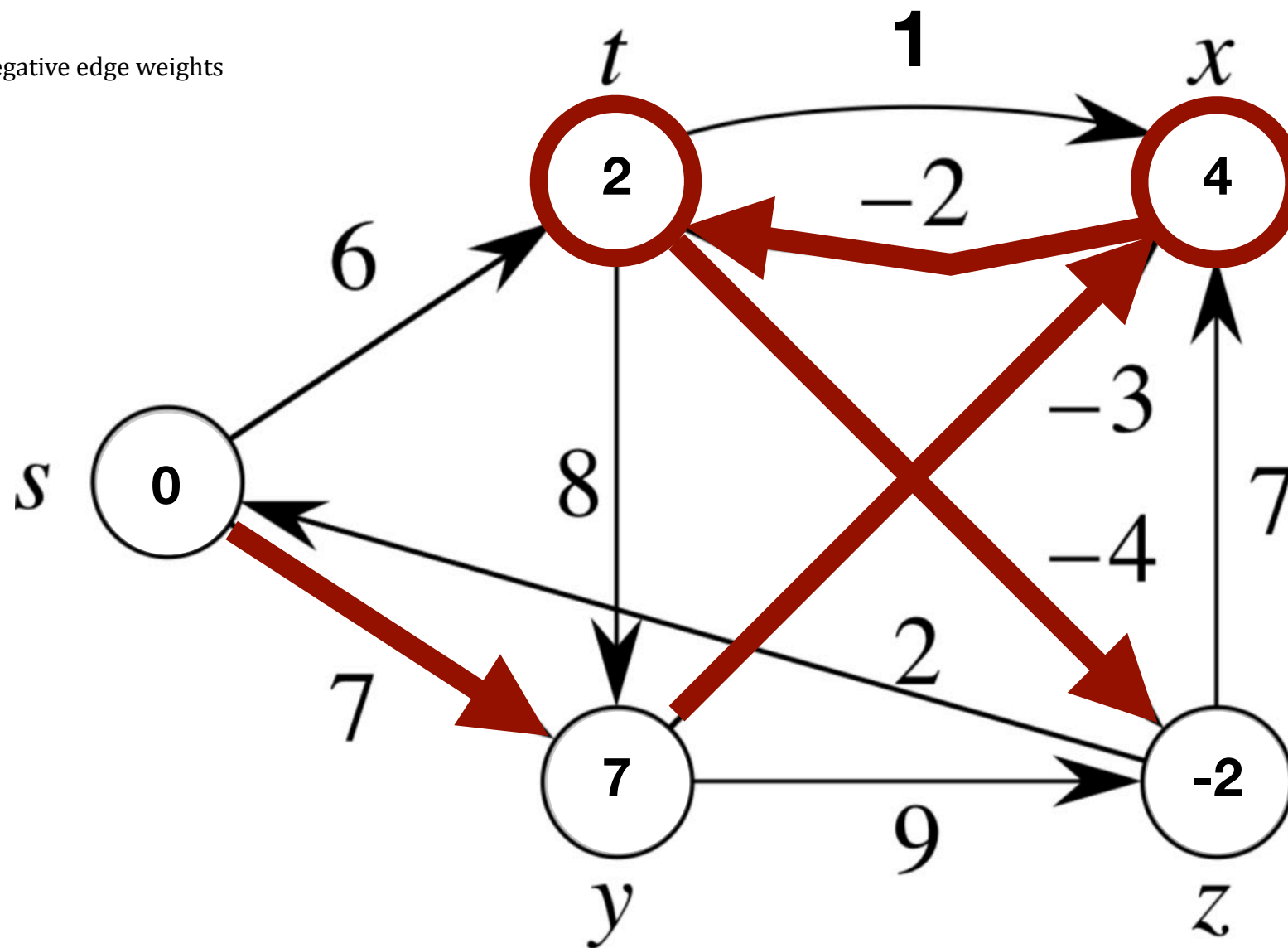
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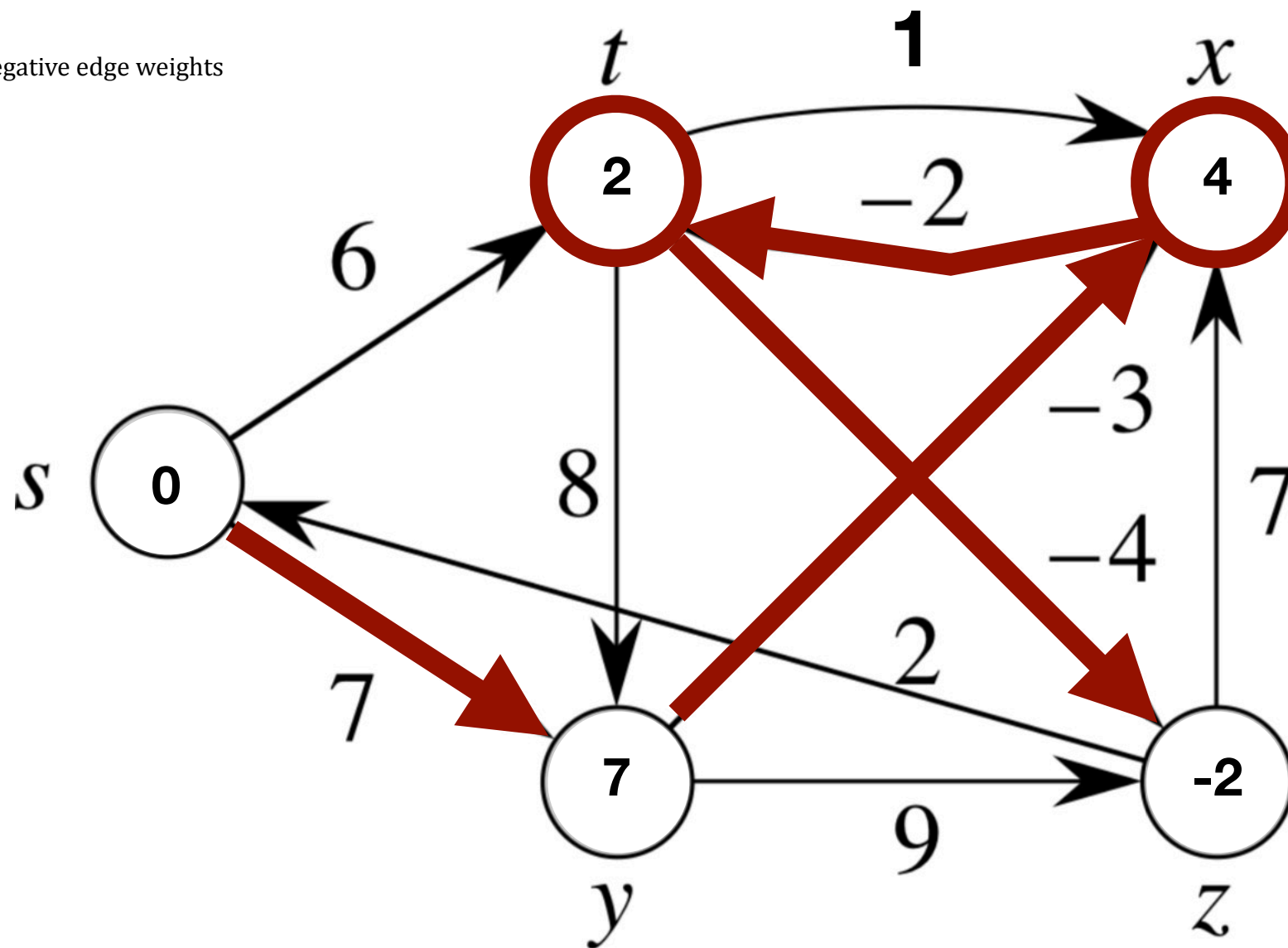
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What does this do?

What if $w(t,x)=1$?

We'd have a negative weight cycle. That's... interesting. Now $w(s,z) = -\infty$.

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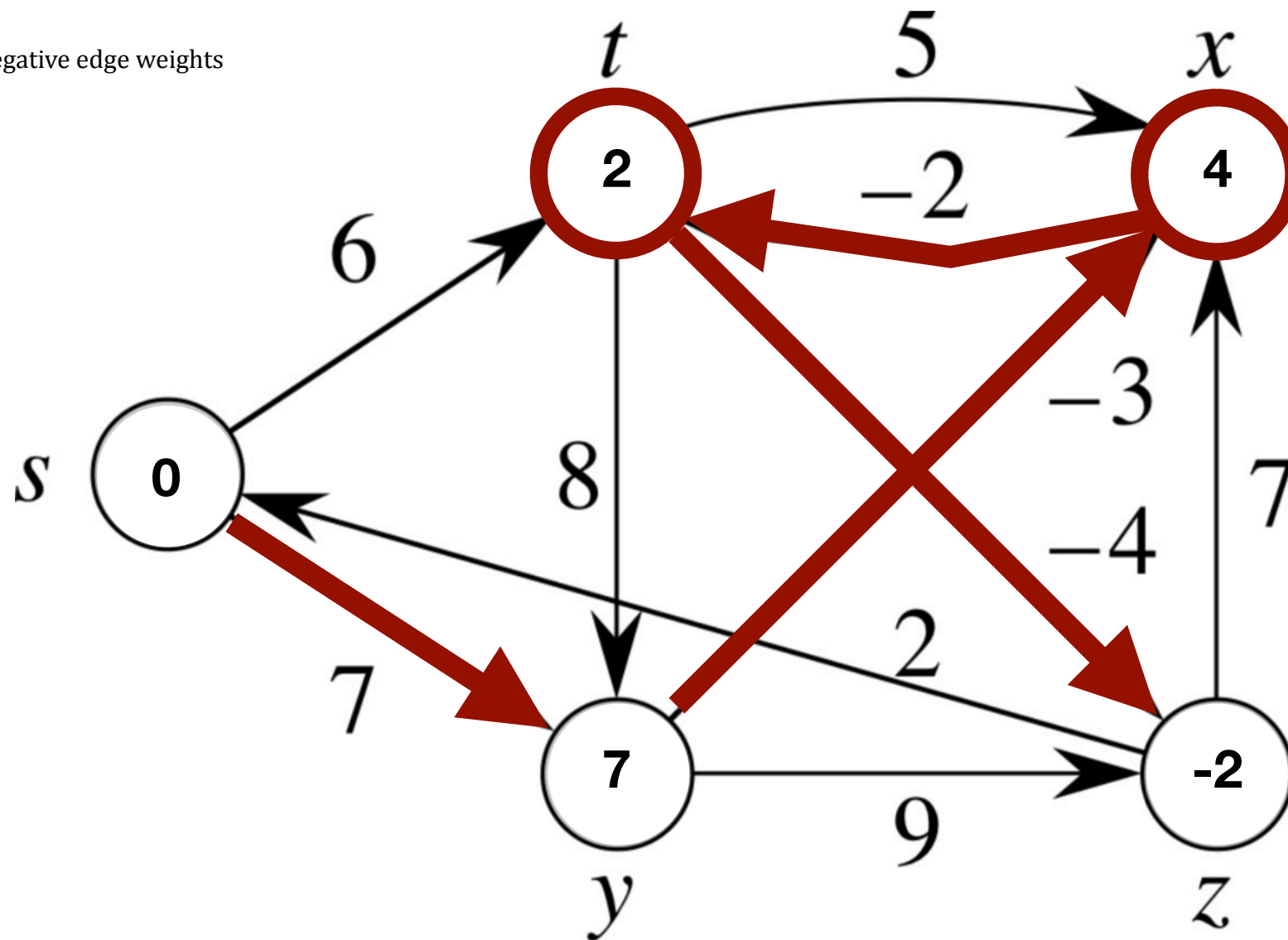
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It detects negative-weight cycles.

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$(x, t : w = -2)$

$(y, x : w = -3)$

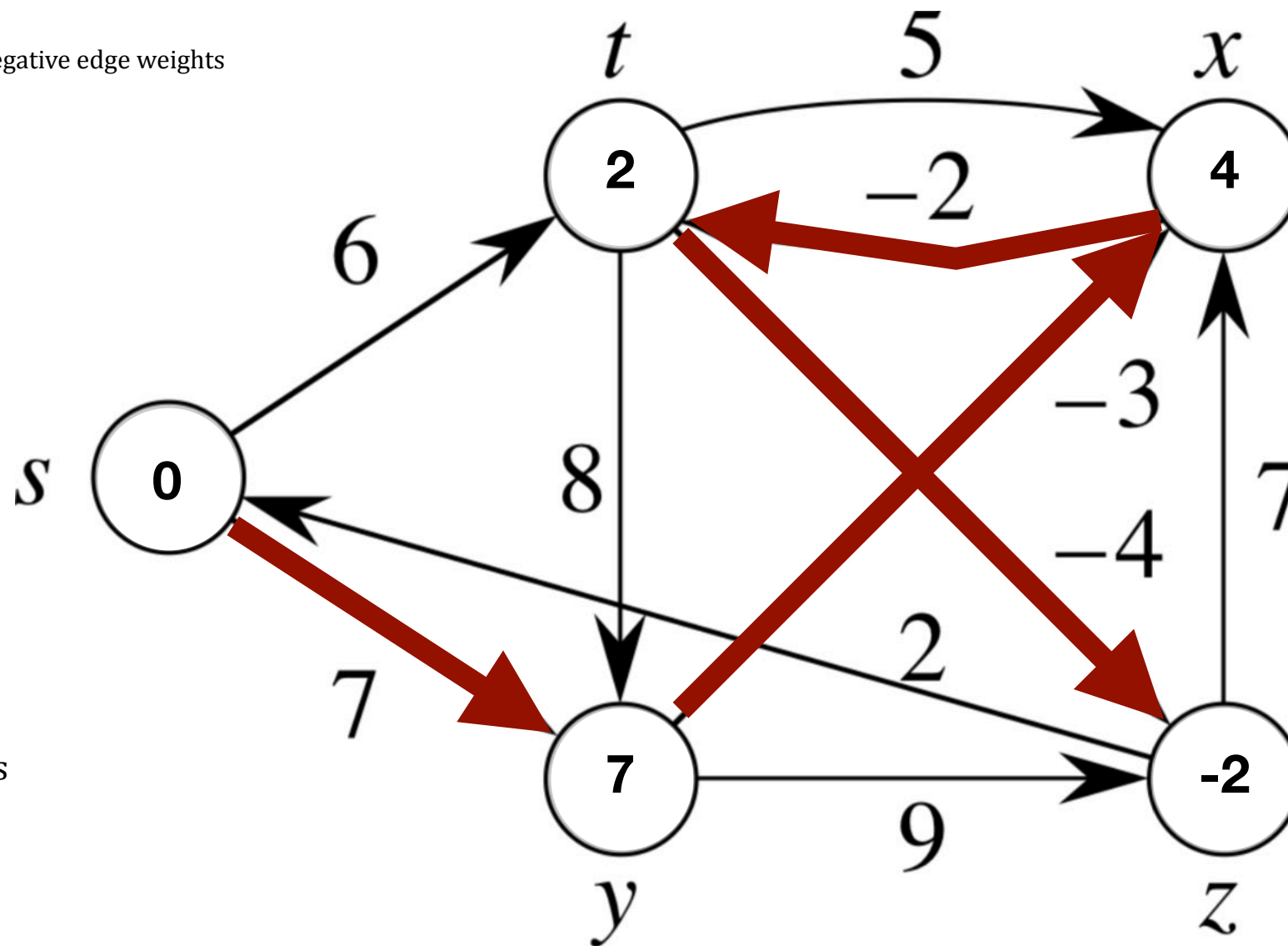
$(y, z : w = 9)$

$(z, x : w = 7)$

$(z, s : w = 2)$

$(s, t : w = 6)$

$(s, y : w = 7)$



Now we're done.

The value in each vertex is the length of the shortest path to that vertex from starting vertex **s**.

$s \rightarrow t = 2$ shortest path: $s \rightarrow y \rightarrow x \rightarrow t$

$s \rightarrow x = 4$ shortest path: $s \rightarrow y \rightarrow x$

$s \rightarrow y = 7$ shortest path: $s \rightarrow y$

$s \rightarrow z = -2$ shortest path: $s \rightarrow y \rightarrow x \rightarrow t \rightarrow z$

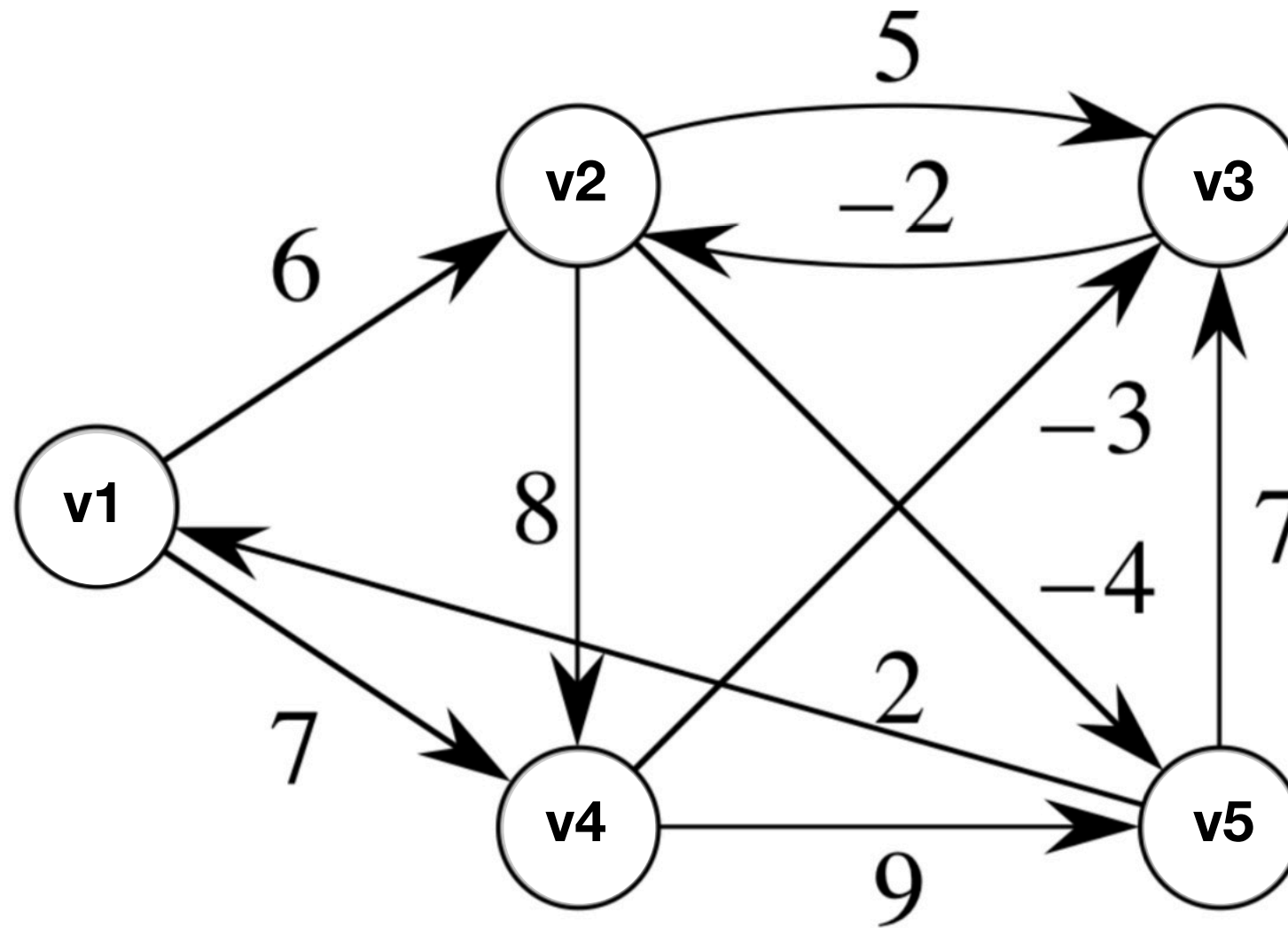
The Bellman-Ford Single-Source Shortest Path Algorithm

Graph G

Assignment 4 version
with integer vertex IDs

new graph

```
add vertex 1
add vertex 2
add vertex 3
add vertex 4
add vertex 5
add edge 2 - 3 5
add edge 2 - 4 8
add edge 2 - 5 -4
add edge 3 - 2 -2
add edge 4 - 3 -3
add edge 4 - 5 9
add edge 5 - 3 7
add edge 5 - 1 2
add edge 1 - 2 6
add edge 1 - 4 7
```



The Bellman-Ford Single-Source Shortest Path Algorithm